1/21/04

2. Homework

Due **1/28/04** before class

1. **Induction (2 points)**
   Prove by induction that

$$\begin{aligned} T(n) &= 1, \text{ if } n = 1 \\ T(n) &= T(n/2) + 1 \text{ if } n \geq 2 \end{aligned}$$

   is in $O(\log n)$. Prove the inductive step and the base case.

2. **Guessing and induction (12 points)**

   For each of the following recurrences find a good guess of what it could solve to (make your guess as tight as possible). You can use either the expansion method or the recursion tree method to find your guess. Then prove that $T(n)$ is in big-Oh of your guess by induction (inductive step and base case). Hint: For simplicity you may want to use $\log_3 n$ instead of $\log_2 n$.

   Every recursion below is stated for $n \geq 2$, and the base case is $T(n) = 1$.
   (a) **(3 points)**
   $T(n) = 3T(\frac{n}{3}) + 1$
   (b) **(3 points)**
   $T(n) = 3T(\frac{n}{3}) + n$
   (c) **(3 points)**
   $T(n) = 3T(\frac{n}{3}) + n \log n$
   (d) **(3 points)**
   $T(n) = 3T(\frac{n}{3}) + n^2$

3. **Recursion tree (2 points)**
   Solve

$$\begin{aligned} T(n) &= 1, \text{ if } n = 1 \\ T(n) &= 2T(n/2) + n^2 \text{ if } n \geq 2 \end{aligned}$$

   using the recursion tree method. You do not need to prove the correctness of your result by induction.

4. **Largest element (4 points)**
   a) Design a divide-and-conquer algorithm to find the position of the largest element in an array of n distinct numbers. (Describe your algorithm in pseudocode with verbal explanation.)
   b) Set up and solve a recurrence relation for the runtime of your algorithm. (You don't need to do an induction. A justification for how you got to your guess is enough.)

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The following exercise has been taken off this homework. It is being postponed until homework 3.

5. **Master theorem (10 points)**
   Use the master theorem to find tight asymptotic bounds for the following recurrences. Justify your results.

   Assume that $T(n)$ is constant for $n \leq 2$.

   - **(2 points)**
     $T(n) = 4T(\frac{n}{3}) + n^4$
   - **(2 points)**
     $T(n) = T(\frac{n}{2}) + \sqrt{n}$
   - **(2 points)**
     $T(n) = T(\frac{7n}{8}) + n$
   - **(2 points)**
     $T(n) = 9T(\frac{n}{3}) + n^2$
   - **(2 points)**
     $T(n) = 5T(\frac{n}{2}) + n^2$