

## Formulas

These are formulas that may be helpful throughout the test.

- **Geometric series:**

$$\sum_{i=0}^n a^i = \frac{a^{n+1} - 1}{a - 1} \quad , \text{if } a \neq 1$$

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1 - a} \quad , \text{if } |a| < 1.$$

- **Arithmetic series:**

$$\sum_{i=1}^n i = \frac{n(n + 1)}{2}$$

- **Master theorem:**

$$T(n) = aT(n/b) + f(n)$$

1. If  $f(n) \in O(n^{\log_b a - \varepsilon})$  for some  $\varepsilon > 0$ , then  $T(n) \in \Theta(n^{\log_b a})$
2. If  $f(n) \in \Theta(n^{\log_b a})$ , then  $T(n) \in \Theta(n^{\log_b a} \log n)$
3. If  $f(n) \in \Omega(n^{\log_b a + \varepsilon})$  for some  $\varepsilon > 0$  and  $af(n/b) \leq cf(n)$  for some  $c < 1$ , then  $T(n) \in \Theta(f(n))$

- **Log rules:**

$$- x = \log_b a \iff b^x = a$$

$$- \log(a * b) = \log a + \log b$$

$$- \log \frac{a}{b} = \log a - \log b$$

$$- \log a^k = k \log a$$

$$- n^{\log_b a} = a^{\log_b n}$$