

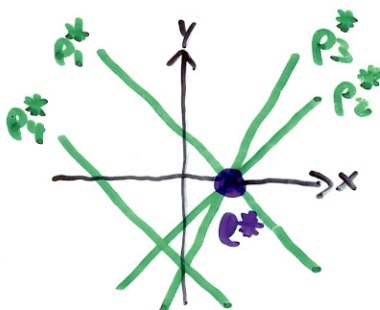
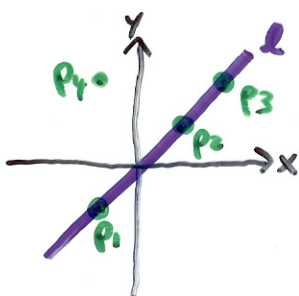
Duality

Points as well as (non-vertical) lines in the plane each have two parameters (x, y -coordinates or slope and intersection with y -axis)

Duality transform: One-to-one mapping of a set of points to a set of lines such that certain properties are preserved.

We consider the following duality transform:

Primal	Dual
Point $p = (p_x, p_y)$	Line $p^* := (y = p_x x - p_y)$
Line $l: y = m x + b$	Point $l^* := (m, -b)$



Properties:

(i) $(p^*)^* = p; (l^*)^* = l$

(ii) $p \in l \Leftrightarrow l^* \in p^*$

incidence preserving

(iii) p lies above l

$\Leftrightarrow l^*$ lies above p^*

order preserving

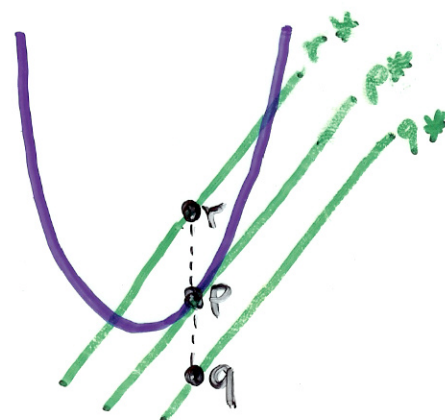
Geometric interpretation: Parabola $\mathcal{U}: y = \frac{x^2}{2}$

• $p \in \mathcal{U} \Rightarrow p^*$ is tangent line at p

• $q \notin \mathcal{U}; p = (q_x, \frac{q_x^2}{2}) \in \mathcal{U}$
 $\Rightarrow q^*$ is parallel to p^*

• $q = (q_x, q_y); r = (q_x, r_y)$

\rightarrow Vertical distance of q^* and r^*
 $= \|q_y - r_y\|$

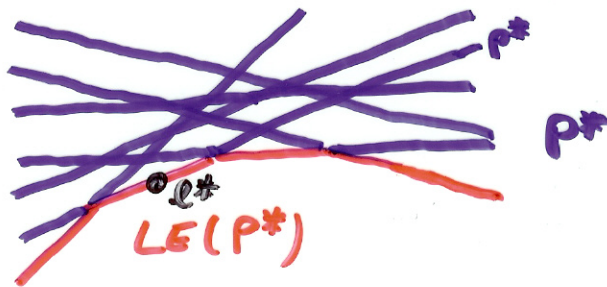
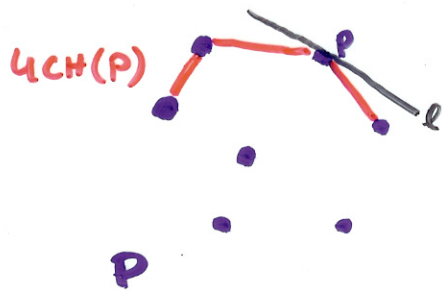


Convex Hulls and Halfspace Intersection

Let $P = \{p_1, \dots, p_n\} \subseteq \mathbb{R}^2$, and let $P^* = \{p_1^*, \dots, p_n^*\}$

Primal plane

Dual plane



Primal: p is a vertex on the upper convex hull $UCH(P)$

\Leftrightarrow there is a non-vertical line l through p such that all other points of P lie below l

Dual: \Leftrightarrow there is a point l^* on the line $p^* \in P^*$ such that l^* lies below all other lines of P^*

$\Leftrightarrow p^*$ contributes an edge to the (unique) bottom cell of the arrangement $A(P^*)$

Lower envelope $LE(P^*)$:= boundary of the bottom cell in $A(P^*)$

- x-monotone polygonal chain
- minimum of linear functions whose graphs are the lines in P^*

boundary of the

- intersection of "lower" halfplanes bounded by a line in P^*

Remarks:

- points on UCH in increasing x-coord. $\hat{=}$ lines of P^* on LE in order of decreasing slope

\rightarrow left-to-right list of points on $UCH \hat{=}$ right-to-left list of lines on LE

- Lower convex hull $\hat{=}$ upper envelope (similar construction)

- Duality cannot handle vertical lines

- No direct correspondence between CH and halfspace intersection (however in projective space)