Efficient Sorting Algorithms

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Specification of the Problem

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a sequence of numbers (or other objects which can be compared to each other and sorted)

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output:
The same numbers in ascending order (i.e., from smallest to largest)

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This algorithm is called Bubblesort.

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Sorting n numbers:

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- etc.
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Sorting n numbers:

- One scan makes $n - 1$ comparisons.
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- etc.
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  $n(n-1)/2$ comparisons.

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This algorithm is called Mergesort.

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1. If the problem size is small enough: solve directly, otherwise:
2. Split the problem into several subproblems of smaller size and solve those recursively.
3. Combine the solutions of the subproblems to one of the complete problem.

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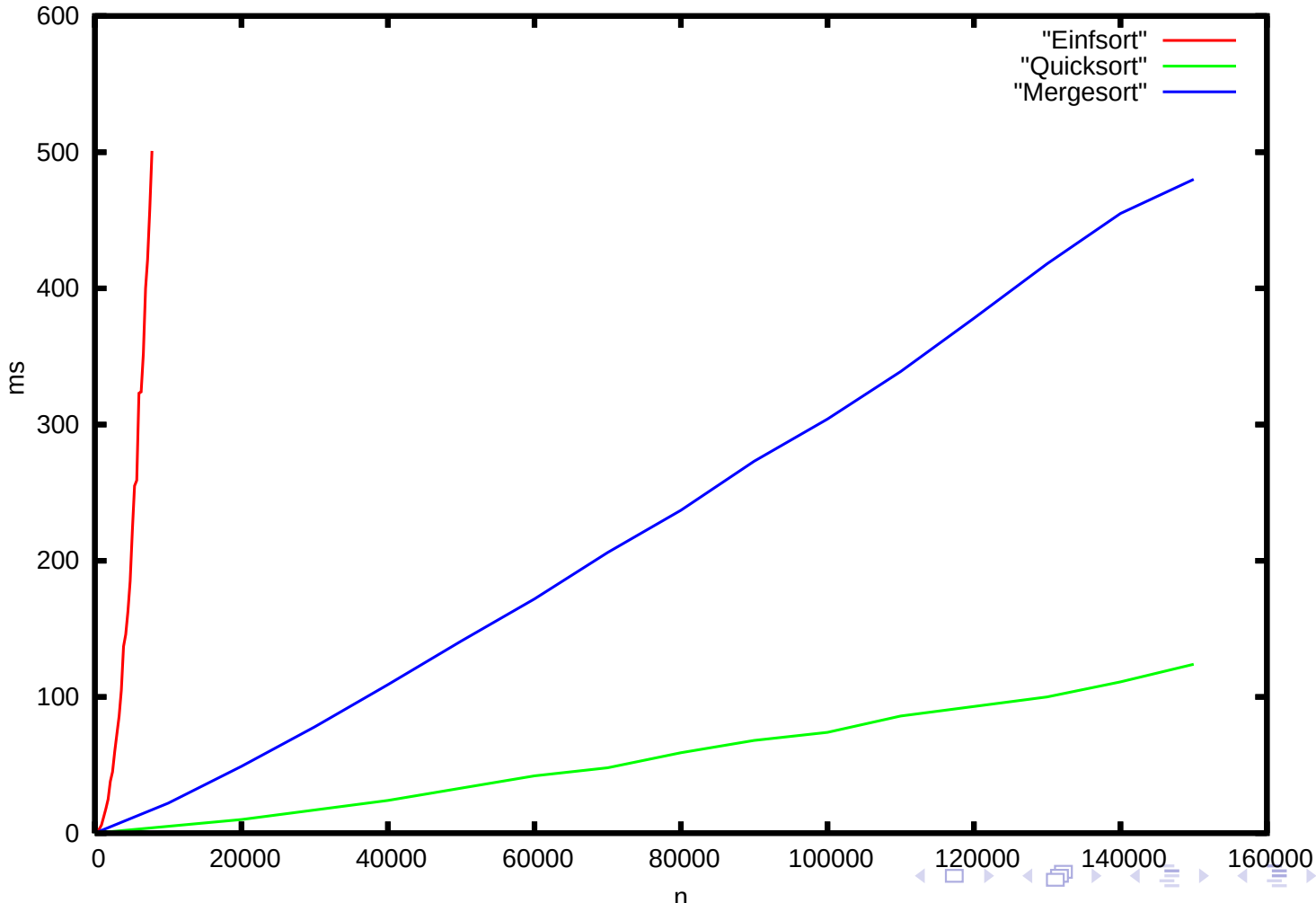
This algorithm is called Quicksort.

Experimental comparison of the sorting algorithms

Runtime of an implementation in milliseconds for different sizes of the input sequence.



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- An algorithm can be formulated in a natural language and even (roughly) analyzed if described in this manner. An explicit program is not necessary: difference between algorithm design and programming.
- There are large differences in the runtimes of different sorting algorithms. A careful runtime analysis already when designing an algorithm makes sense.

Links

- **Visual comparison of sorting algorithms**
  `http://www.sorting-algorithms.com/`
- **Acoustic demonstration of sorting algorithms**
  `http://www.youtube.com/watch?v=kPRA0W1kECg`
- **Sorting algorithms demonstrated by folk dances**
  - **Bubblesort**
    `http://www.youtube.com/watch?v=lyZQPjUT5B4`
  - **Mergesort**
    `http://www.youtube.com/watch?v=XaqR3G_NVoo`
  - **Quicksort**
    `http://www.youtube.com/watch?v=ywWBy6J5gz8`
- **Demo in Scratch**
  `http://scratch.mit.edu/projects/682862/`

Homework

1. Sort the following sequence of 3-letter words *alphabetically* by Bubblesort, Mergesort, and Quicksort:
   JAN, FEB, MAR, APR, MAY, JUN, JUL, AUG, SEP, OCT, NOV, DEC.
2. Give a recursive algorithm for the *selection* problem specified as follows:
   Given an unsorted sequence of numbers (or other comparable elements) and a number k, *find* the k-th smallest element in the sequence, i.e., the one that occurs at the k-th position from the left, if the input sequence is sorted.
   For example, the 4th smallest element in the sequence of exercise 1 would be FEB.
   Don't use sorting, but find a more efficient algorithm.
   Demonstrate your algorithm with an example.
   Hint: The idea of splitting the input sequence as in Quicksort may be used, then one recursive call to a smaller subsequence gives the result.