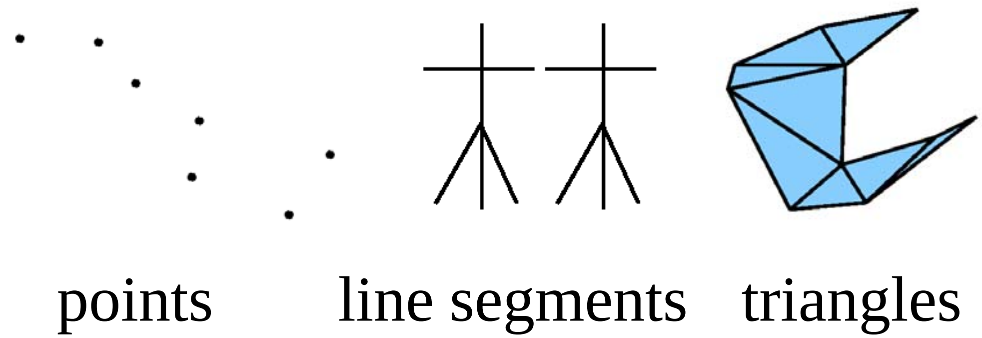
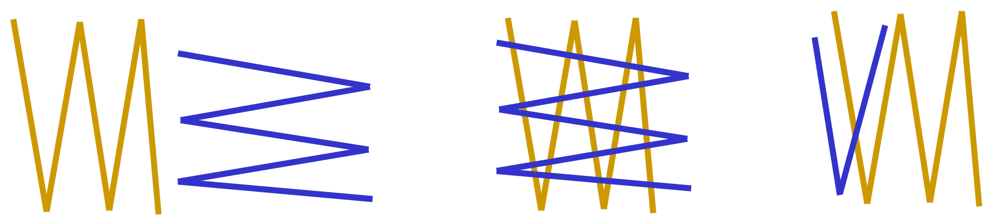
Geometric Shape Matching

- Consider **geometric shapes** to be composed of a number of basic objects such as



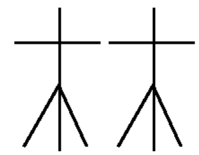
- How **similar** are two geometric shapes?

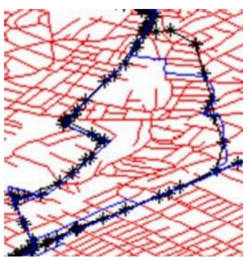
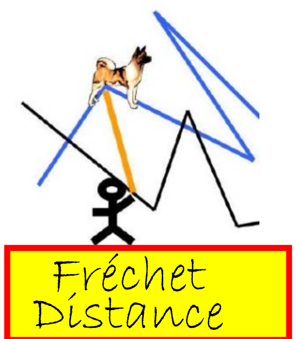


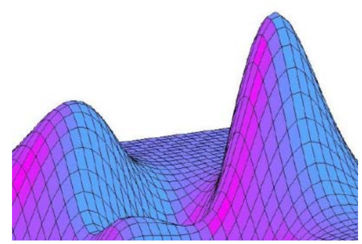
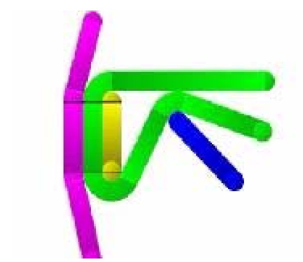
- Transformations (translations, rotations, scalings)
- Choice of distance measure
- Full or partial matching

Research Interests

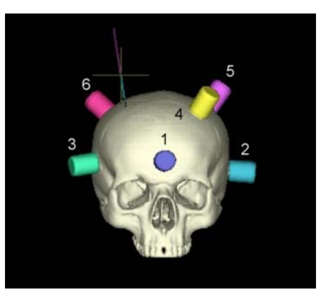
- Computational Geometry:
  Shape matching

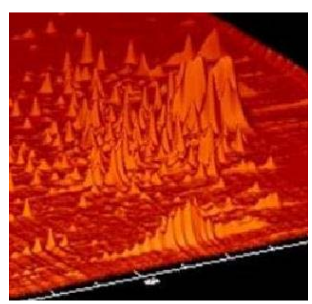
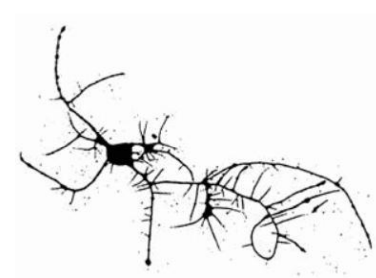



- Biomedical applications

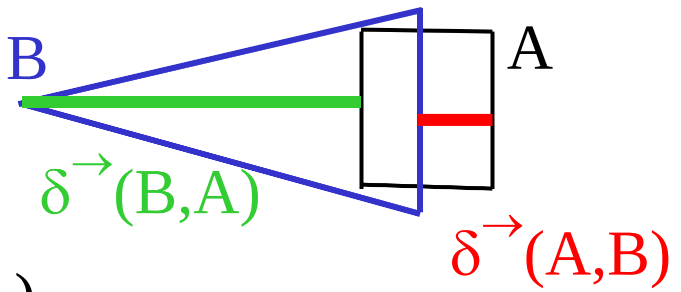


When are curves „similar“?

- Directed Hausdorff distance

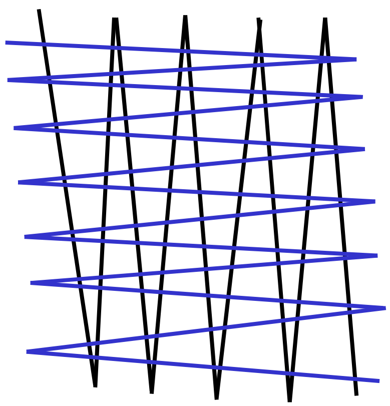
  $\delta^{\rightarrow}(A,B) = \max \min \| a-b \|$



- Undirected Hausdorff distance

  $\delta(A,B) = \max (\delta^{\rightarrow}(A,B) , \delta^{\rightarrow}(B,A))$

But:

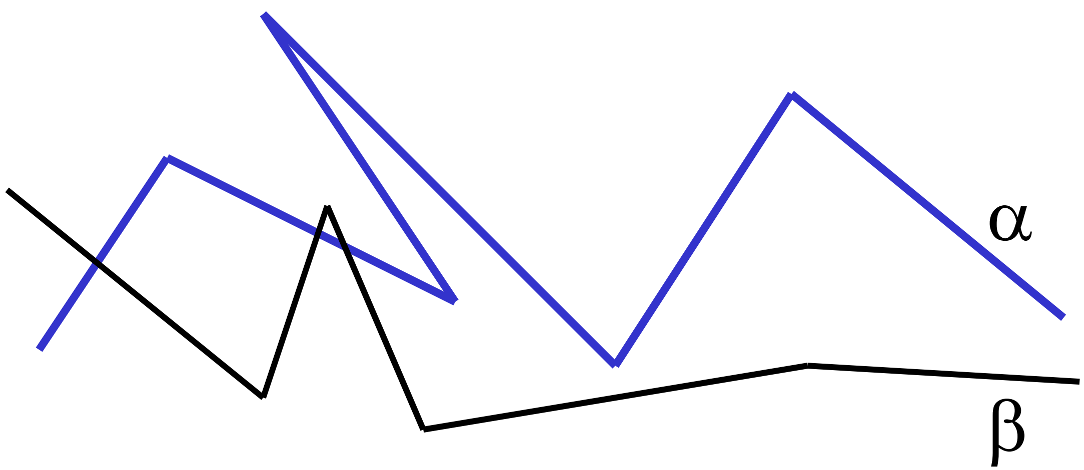


- Small Hausdorff distance
- When considered as curves the distance should be large
- The Fréchet distance takes continuity of curves into account

Fréchet Distance

$$\delta_F(\alpha,\beta) = \inf_{f,g:[0,1]\to[0,1]} \max_{t\in[0,1]} \|\alpha(f(t))-\beta(g(t))\|$$

where f and g range over continuous non-decreasing reparametrizations only.



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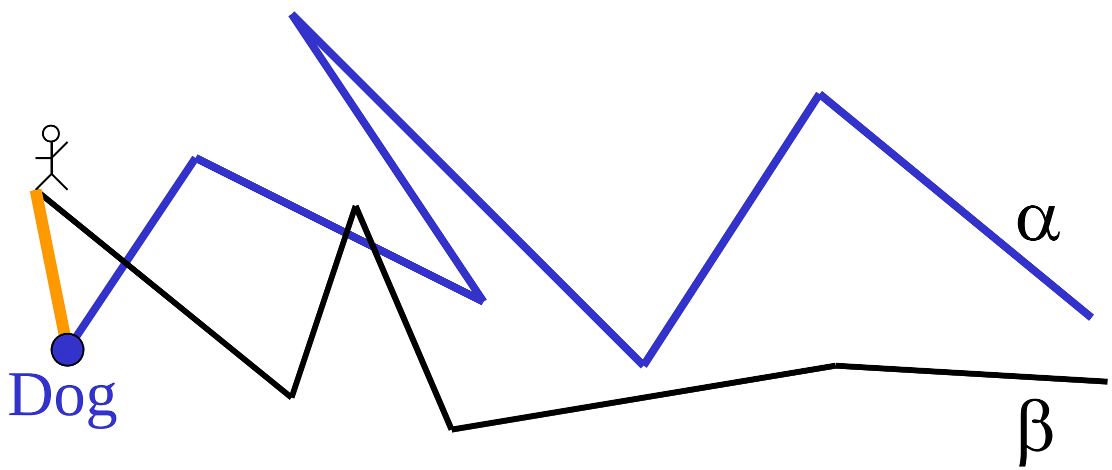

- Man and dog walk on one curve each
- They hold a **leash**

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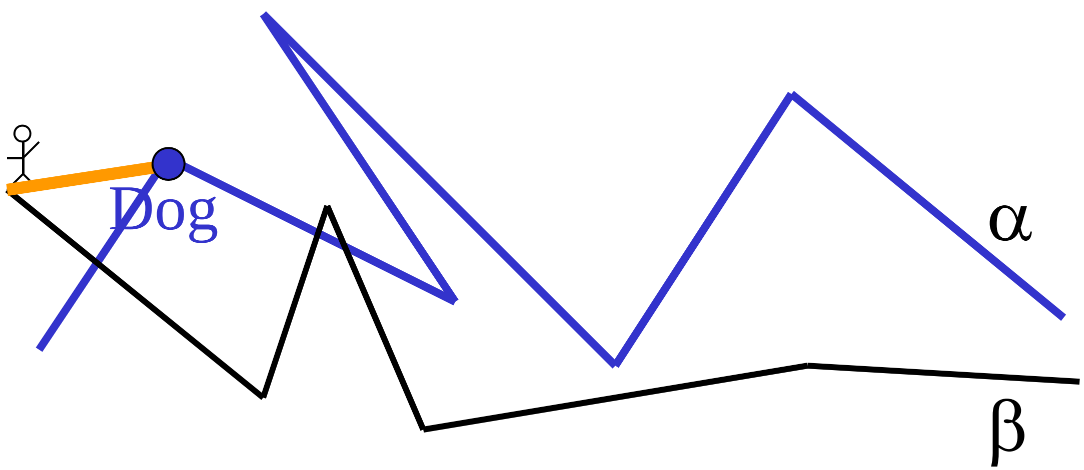

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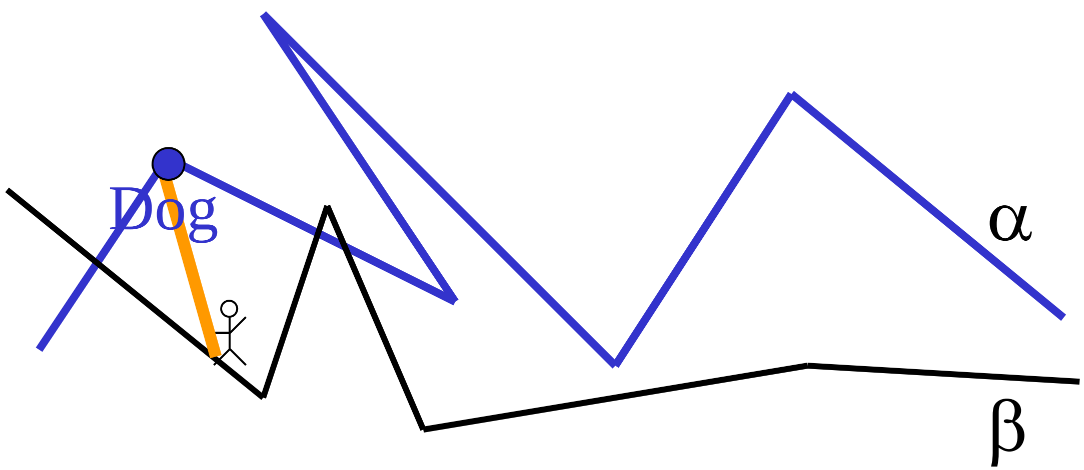

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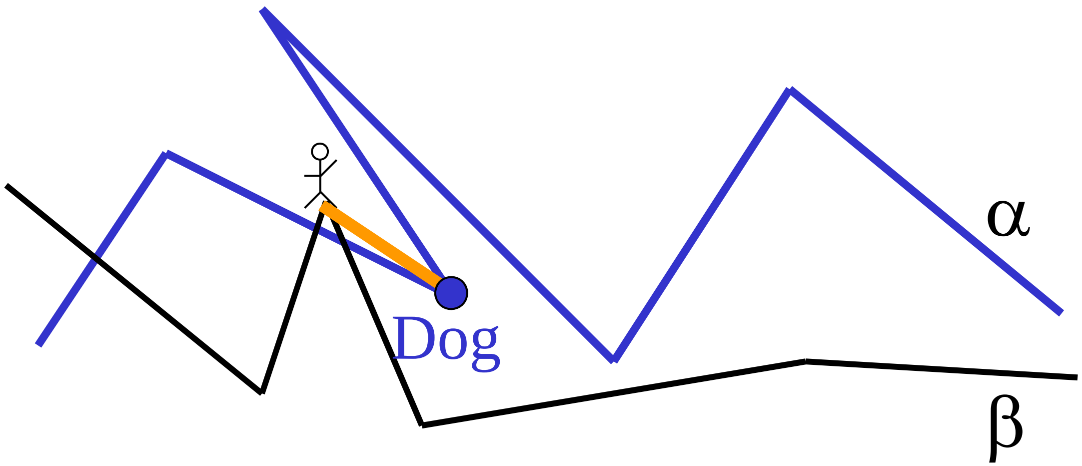

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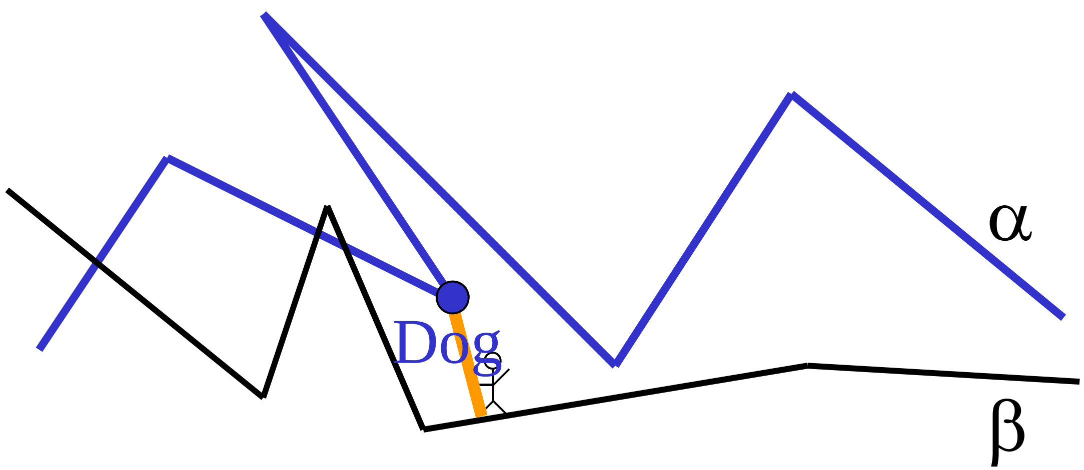

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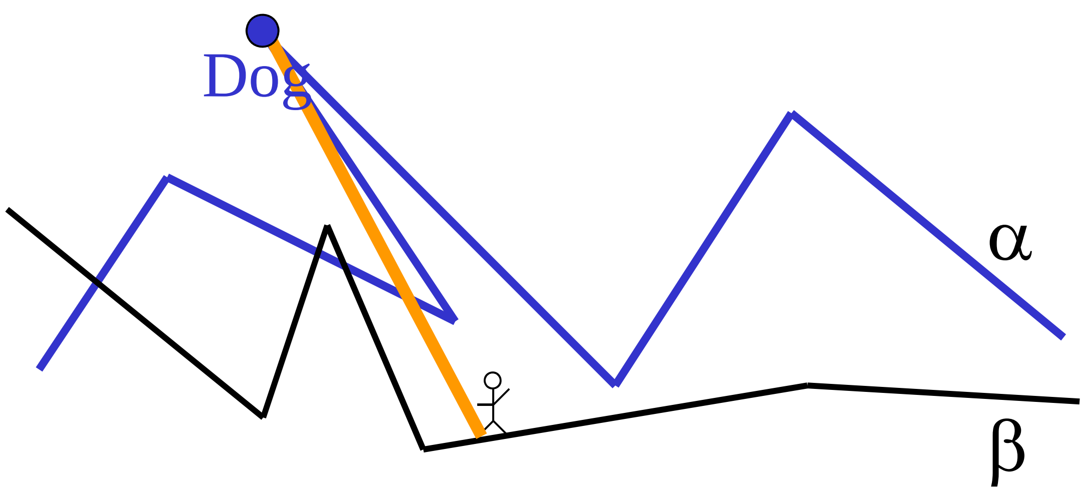

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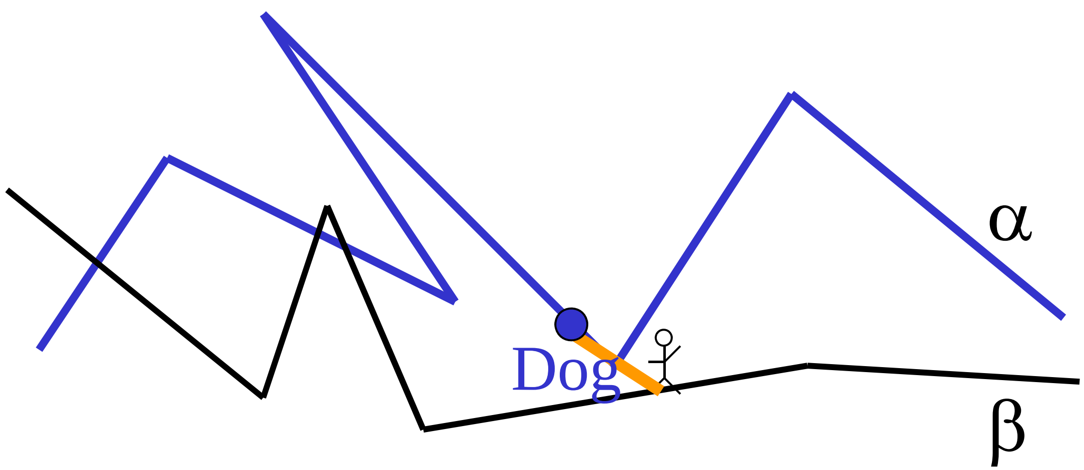

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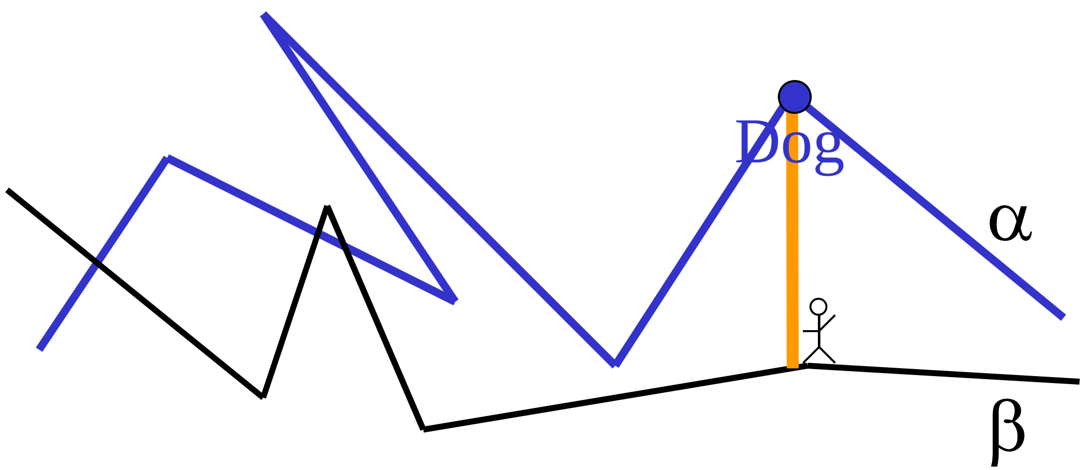

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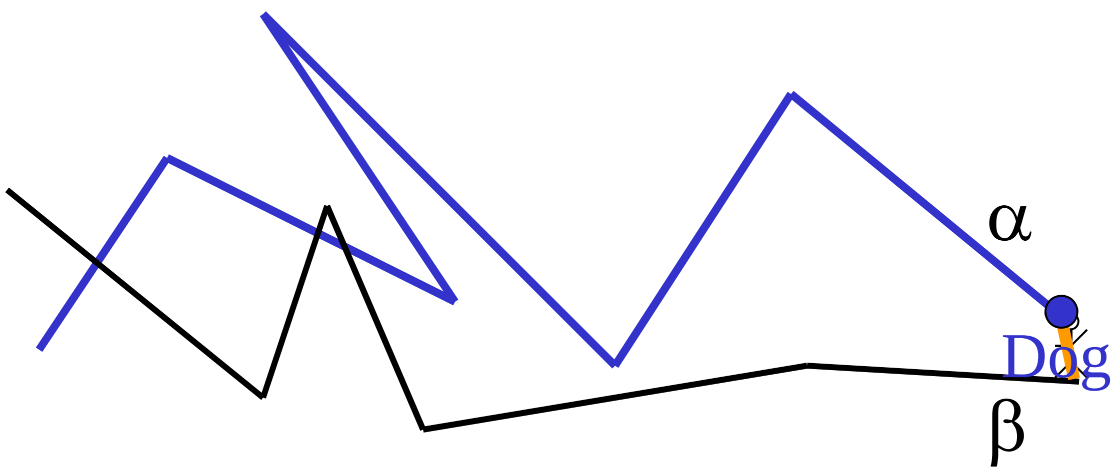
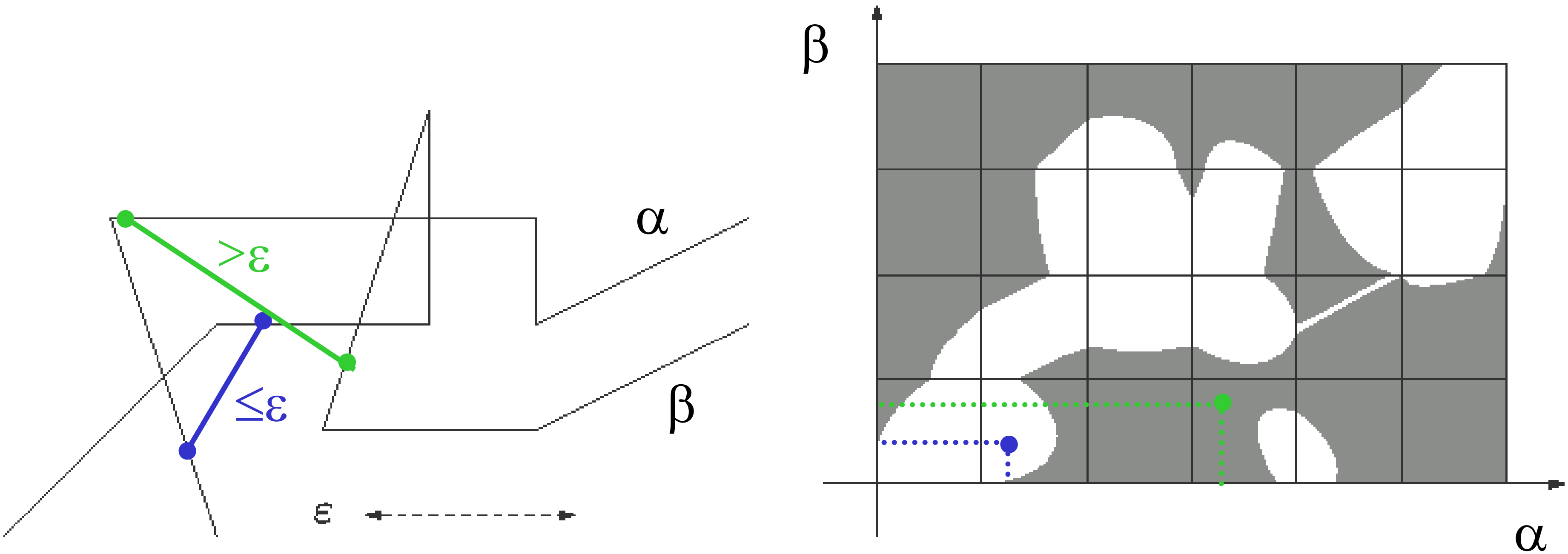


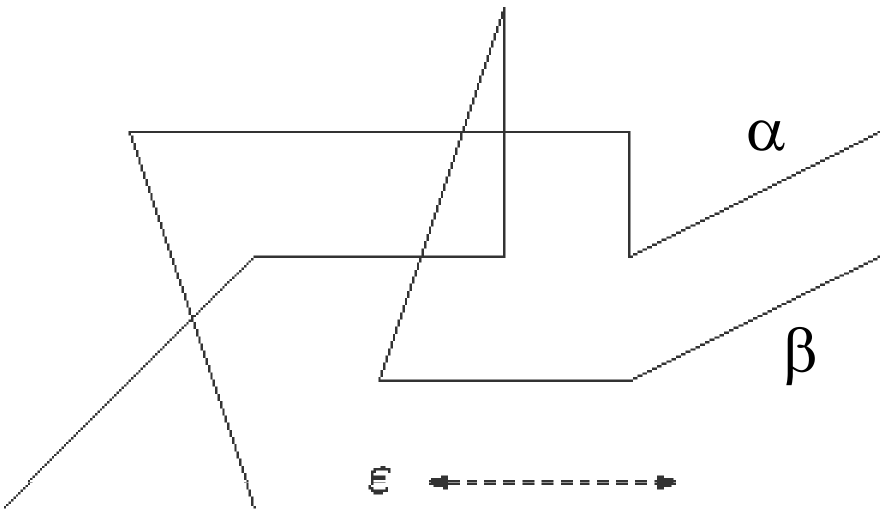
- Man and dog walk on one curve each
- They hold a **leash**
- Only allowed to go forward
- δ_F is the minimum possible leash length

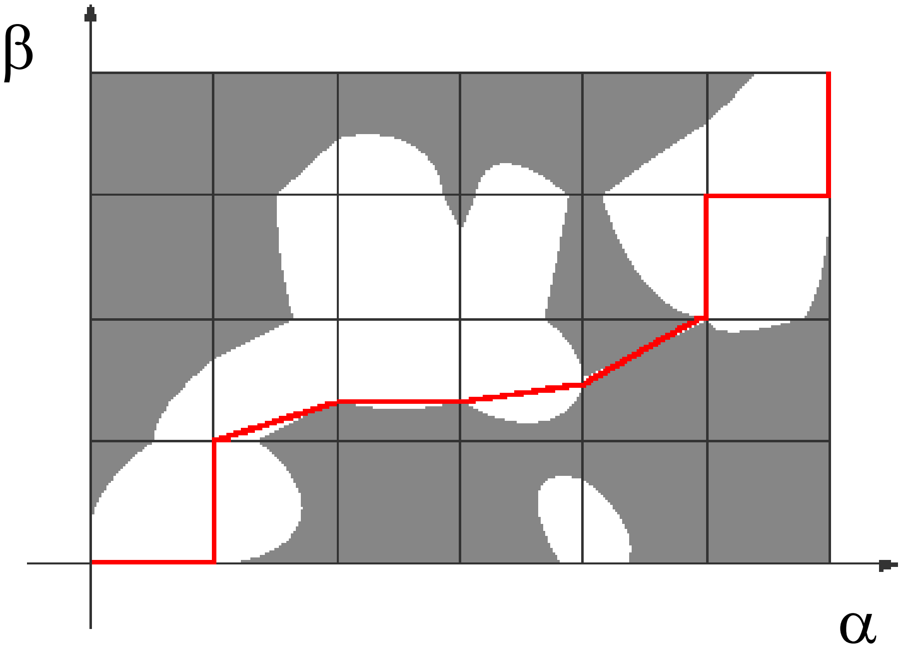
Free Space Diagram



- Let $\varepsilon > 0$ fixed (eventually solve decision problem)
- $F_\varepsilon(\alpha,\beta) = \{ (s,t) \in [0,1]^2 \mid \| \alpha(s) - \beta(t) \| \leq \varepsilon \}$ *white points*
  free space of α and β
- $FD_\varepsilon(\alpha,\beta)$ **free space diagram** *white and black points*

Free Space Diagram





- $\delta_F(\alpha,\beta) \leq \varepsilon$ iff there is a monotone path in the free space from (0,0) to (1,1)
- Monotone path encodes reparametrizations of α and β
- Find monotone path in O(*mn*) time