CS 6463 Computational Geometry, Fall 10

11/10/10

5. Homework

Due **11/24/10** before class

For all problems below, analyze the space complexity, the preprocessing time, and the query time.

1. Smallest Rectangle Queries (10 points)
   Let P be a set of n points in the plane; you may assume that they are in general position. Devise a data structure of size $O(n \log n)$ to answer queries of the following form in $O(\log^2 n)$ time:

   > Given a vertical line segment s and an integer k, find the smallest rectangle that has s as its left side and which contains at least k points. If no such rectangle exists then indicate this.

2. Triangular Range Query (10 points)
   Let P be a set of n points in the plane. Devise a data structure of size $O(n)$ to answer queries of the following form in $O(n^{2/3} + k)$ time, where k is the number of points reported:

   > Given a right triangle T in which one side is parallel to the x-axis, another side parallel to the y-axis, and the third side has a slope of -1. Report all the points of P that lie within T.

   Hint: Transform this problem into an orthogonal range search problem in dimension 3.

3. Nesting segment trees and range trees (10 points)
   In class we used a *segment-range* tree to solve the 2-dimensional windowing problem. This two-level tree consists of a segment tree as the primary tree, and it stores in each node of the primary tree a link to a secondary tree which is implemented as a range tree.

   Now consider defining a *range-segment* tree which has a range tree as the primary tree and segment trees as the secondary trees. We can also define a *segment-segment* tree in a similar way, and *range-range* trees we have already studied in class.

   Compare all four data structures and argue what kinds of problems each can be used to solve. Analyze and compare the query times, construction times, and space complexities.