## CS 6463 Computational Geometry, Fall 10

## 3. Homework <br> Due 10/20/10 before class

1. Parabolic Arcs (5 points)

Give an example where the parabola defined by some site $p_{i}$ contributes more than one arc to the beach line. Can you give an example where it contributes a linear number of arcs?
2. Sum of Edge Lengths (5 points)

Give an example which shows that the Delaunay Triangulation of a point set is not always the triangulation with the minimum sum of edge lengths.
3. Weighted Voronoi Diagrams (10 points)

Let $P=\left\{p_{1}, \ldots, p_{n}\right\} \subseteq \mathbb{R}^{2}$, and let $w_{i}>0$ be the weight of point site $p_{i}$, for each $i=1, \ldots, n$. In the additively weighted Voronoi diagram the Voronoi cell for $p_{i}$ is defined as

$$
V_{a d d}\left(p_{i}\right)=\left\{q \in \mathbb{R}^{2} \mid w_{i}+\left\|p_{i}-q\right\|<w_{j}+\left\|p_{j}-q\right\| \text { for all } p_{j} \in P \backslash\left\{p_{i}\right\}\right\}
$$

In the multiplicatively weighted Voronoi diagram the Voronoi cell for $p_{i}$ is defined as

$$
V_{\text {mult }}\left(p_{i}\right)=\left\{q \in \mathbb{R}^{2} \mid w_{i} *\left\|p_{i}-q\right\|<w_{j} *\left\|p_{j}-q\right\| \text { for all } p_{j} \in P \backslash\left\{p_{i}\right\}\right\}
$$

Show how the bisectors look like for both kinds of weighted Voronoi diagrams, and give some examples of Voronoi diagrams for each case. You are welcome to research on the web as long as you give references.
4. Edge Flips (10 points)
(a) Show that any two triangulations of a convex polygon can be transformed into each other by edge flips.
(b) Show that any two triangulations of a planar point set can be transformed into each other by edge flips.
5. Worst-Case DT Runtime (5 points)

Show that the worst-case runtime of the randomized algorithm to compute the Delaunay triangulation of a set of $n$ points in the plane is $\Omega\left(n^{2}\right)$. Hint: Find $a$ worst-case example using one of the Delaunay triangulation programs.

