9/8/10

1. Homework Due 9/22/10 before class

Always justify the runtime and the correctness of your algorithms.

1. Convex Hulls (15 points)

This exercise is about computing the convex hull of objects other than points.

- (a) Let S be a set of n line segments in the plane. Prove that the convex hull of S is exactly the same as the convex hull of the 2n endpoints of the segments. Hint: It is enough to use the definition of the convex hull as the intersection of convex sets.
- (b) Let \mathcal{P} be a non-convex polygon. Describe an algorithm that computes the convex hull of \mathcal{P} in O(n) time. *Hint: Use a variant of Graham's scan where the vertices are not treated in lexicographic order but in some other order.*

2. Line Segment Intersection (5 points)

Let $a, b, c, d \in \mathbb{R}^2$. Show how to check whether the two line segments ab and cd intersect using a constant number of half-plane tests (or orientation tests).

3. Visible Segments (10 points)

Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that P can see, i.e., all line segments of S that contain some point q so that the open segment pq = 1 does not intersect any line segment of S.

Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p.



4. Guarding the Boundary vs. the Interior (5 points)

Give an example of a polygon together with a placement of vertex guards, such that the whole polygon boundary is guarded but not the whole interior.