CS 6463-004 Computational Geometry, Fall 06

11/9/06

5. Homework

Due **11/30/06** before class

Always justify the runtime and the correctness of your algorithms, and try to make algorithms as efficient as possible.

1. Smallest Rectangle Queries (10 points)
   Let P be a set of n points in the plane; you may assume that they are in general position. Devise a data structure of $O(n \log n)$ size to answer queries of the following form in $O(\log^2 n)$ time: Given a vertical line segment s and an integer k, find the smallest rectangle that has s as its left side and which contains at least k points. If no such rectangle exists then indicate this. (Assume that there exists a data structure based on range trees for answering orthogonal range *counting* queries in dimension d in $O(\log^{d-1} n)$ time with $O(n \log^{d-1} n)$ space.)

2. (5.9) Point Range Queries (10 points)
   One can use the kd-tree and the range tree on a set S of points in the plane to determine whether a particular point (a, b) is in S by performing a range query with range $[a, a] \times [b, b]$.

   (a) Prove that performing such a range query on a kd-tree takes time $O(\log n)$.

   (b) What is the time bound for such a query on a range tree? Prove your answer.

3. Triangular Range Query (10 points)
   Let P be a set of n points in the plane. Devise a data structure of $O(n)$ size to answer queries of the following form n $O(n^{2/3} + k)$ time, where k is the number of points reported: Given a right triangle T in which one side is parallel to the x-axis, another side parallel to the y-axis, and the third side has a slope of -1. Report all the points of P that lie within T. *Hint: Transform this problem into an orthogonal range search problem in dimension 3.*

4. Ray shooting (10 points)
   Let S be a set of n disjoint line segments in the plane. Describe a data structure that can preprocess S in $O(n \log n)$ time and space such that the following *ray shooting* query can be answered in $O(\log n^2)$ time: Given a point $q = (q_x, q_y)$ report the first segment hit by a vertical ray starting at q and going vertically to infinity.

X. (5.13) **Extra Credit** (10 points)
   This problem is for extra credit and not mandatory.

   a) Let S be a set of n axis-parallel rectangles in the plane. We want to be able to report all rectangles in S that are completely contained in a query rectangle $[x, x'] \times [y, y']$. Describe a data structure for this problem that uses $O(n \log^3 n)$ storage and has $O(\log^4 n + k)$ query time, where k is the number of reported answers. *Hint: Transform the problem to an orthogonal range searching problem in some higher-dimensional space.*

   b) Same as in a), but this time S consists of n polygons in the plane with total complexity n.