CS 5633 Analysis of Algorithms – Spring 12

2/21/12

5. Homework

Due **3/1/12** before class

1. **Worst-case example for insertion into a red-black tree**
   Give an example of a red-black tree of height $h \geq 5$ as well as a key, such that the red-black tree procedure for inserting the key makes the maximum possible number of case-1 color changes.

2. **Transforming binary search trees**
   (a) Describe an algorithm that uses $O(n)$ rotations to transform a binary search tree that stores n keys into a degenerate left-chain binary search tree that stores the same n keys.
   (b) Describe an algorithm that transforms a binary search tree T_1 that stores n keys into another binary search tree T_2 that stores the same n keys, using $O(n)$ rotations.

3. **B-trees**
   (a) What is the maximum number of keys that can be stored in a B-tree with minimum degree k and height h? Your answer should depend on k and h.
   (b) Show that if B-Tree-Search is changed to use binary search instead of linear search on the key, then the CPU time is $O(\log n)$, which is independent of k.

4. **Interval trees**

   The interval-search procedure covered in class returns an interval that overlaps the query, but there could be many such intervals overlapping the query. The goal of this problem is to modify the interval-search procedure to output the interval that has the minimum low endpoint.

   (a) Give an example interval tree and a query, for which the interval-search procedure returns an interval that **does not have** the minimum low endpoint among all intervals that overlap the query.
   (b) Describe a different interval-search procedure which returns an interval that **has** the minimum low endpoint among all intervals stored in the tree that overlap the query.