CS 5633 Analysis of Algorithms – Spring 12

4/10/12

10. Homework

Due **4/19/12** before class

1. **Dijkstra variants**
   (a) Let G be a directed graph with non-negative edge weights. Consider changing the condition in the while loop of Dijkstra's algorithm to $|Q| > 1$. Prove that at the end of the algorithm the d-values contain the shortest path weights (so, the shortest path weights are correctly computed even when skipping the last iteration from Dijkstra's algorithm).
   (b) Let G be a directed graph with non-negative edge weights. Consider changing the condition in the while loop of Dijkstra's algorithm to $|Q| > 2$. Show that at the end of the algorithm the d-values do not necessarily contain the shortest path weights (give a counter-example).
   (c) Now, let G be a directed graph with arbitrary edge weights, and assume that some of the edge weights are negative. Let W be the smallest (negative) weight of any edge. Consider reweighing each edge weight by adding $-W$ to every edge, which yields a graph G' with non-negative edge weights. Then run Dijkstra's algorithm on G', and in the end add W to every computed d-value. Give a counter-example to show that this approach does not always compute the correct d-values.

2. **Floyd-Warshall**
   (a) Show how to use the Floyd Warshall algorithm to detect whether a weighted graph contains a negative weight cycle.
   (b) Suppose you run the Floyd Warshall algorithm for $k = 1$ to $n - 1$, and not to n. Does this still compute the correct output? Justify your answer.
   (c) In the Floyd Warshall algorithm, can you switch the order of the three for-loops and still compute the correct output? Justify your answer.

3. **P or NP?**

   Which of the problems below are in P or not? Which of the problems below are in NP or not? Justify your answers.

   (a) Given an array A of n numbers. Does A have the structure of a min-heap?
   (b) Compute a heap from an array A of n numbers.
   (c) Given an array A of n numbers, and a number k. Does A contain the number k?
   (d) Given an array A of n numbers, and a number k, is there a subset of numbers in A that sum up to exactly k?

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(e) Given a directed graph $G = (V, E)$. Is G a DAG?

(f) Given an undirected edge-weighted graph $G = (V, E)$, compute a MST.

(g) Given two undirected graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$. Is H an isomorphic subgraph of G? (Intuitively, this means that G contains a "copy" of H. More formally, there is a one-to-one correspondence between all vertices in V_H and a subset V' of vertices of V_G such that each edge in E_H corresponds to an edge between vertices in V'.)

4. **Reductions**

Let Π, Π', Π'' be three decision problems. Use the definitions of P, NP, and the polynomial-time reduction " $\leq''$, to prove the following three facts (from slide 17):

(a) If $\Pi \in P$ and $\Pi' \leq \Pi$ then $\Pi' \in P$.

(b) If $\Pi \in NP$ and $\Pi' \leq \Pi$ then $\Pi' \in NP$.

(c) If $\Pi \leq \Pi'$ and $\Pi' \leq \Pi''$ then $\Pi \leq \Pi''$.