CS 5633 Analysis of Algorithms – Spring 10

4/13/10

9. Homework

Due **4/20/10** before class

1. **Floyd-Warshall in less space (4 points)**
   Show how Floyd-Warshall's algorithm can be implemented to use only $\Theta(n^2)$ space (see problem 25.2-4 on page 634 / 699 in the book).

2. **Negative-weight cycle (5 points)**
   Given a directed weighted connected graph $G = (V, E)$ with **real** edge weights (i.e., **negative** edge weights are allowed). Give an algorithm (in words is enough, but if you need to you can write pseudo-code) that detects **AND prints** out a negative-weight cycle if G contains a negative-weight cycle. What is the runtime of your algorithm?

3. **Floyd-Warshall (4 points)**
   During the Floyd-Warshall all-pairs shortest paths algorithm, the shortest paths can be stored in a predecessor matrix. This is similar to storing a predecessor array for Dijkstra's algorithm, just that there is such an array for every vertex. (Page 632 / 695 in the textbook covers this topic, however it is possible to express the formula in a simpler way.)

   (a) (2 points) Modify Floyd-Warshall's algorithm to include the computation of the predecessor matrix.

   (b) (2 points) Write a method to use the predecessor matrix to print a shortest path between two vertices i and j.

4. **Transitivity (3 points)**

   Show the transitivity property of the polynomial-time reduction "$\leq$" (fact 3 on slide 17):
   Let Π, Π', Π'' be three decision problems. If $\Pi \leq \Pi'$ and $\Pi' \leq \Pi''$ then $\Pi \leq \Pi''$.

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5. **To be or not to be... ... in NP (5 points)**
Which of the problems below are in NP and which are not? Justify your answers.

(a) Given an unsorted array A of n numbers, and a number k. Does A contain the number k?

(b) Given an unsorted array A of n numbers. What is the minimum of the numbers stored in A?

(c) Given an undirected graph G. Is G a tree?

(d) Given a connected directed graph $G = (V, E)$ and a number $k > 0$. Is G k-colorable? (A graph is k-colorable if there exists an assignment of at most k colors to vertices, one color per vertex, such that no two vertices that share the same edge have the same color.)

(e) Given n numbers. Can these numbers be partitioned into two (disjoint) sets A, B such that the sum of the numbers in A equals the sum of the numbers in B?