CS 5633 Analysis of Algorithms – Spring 10

1/14/10

1. Homework

Due **1/21/10** before class

1. **Code snippets (5 points)**

   For each of the two code snippets below give their Θ-runtime depending on n. Justify your answers.

   (a) **(3 points)**

   ```
   for(i=1; i<=n; i=i*3){
     for(j=n; j>=1; j=j/3){
         for(l=2; l<=n; l=l*l*l){
           print(" ");
         }
       }
     }
   }
   ```

   (b) **(2 point)**

   ```
   for(i=n/2; i>=1; i=i-1){
     for(j=i; j<=n; j=j+i){
       print(" ");
     }
   }
   ```

2. **Big-Oh (2 points)**
   Prove the following, using the definitions of Big-Oh:
   $n^3 + 2n + 1 \notin O(n^2)$

3. **Theta (2 points)**
   Use the definition of Θ to prove the following:
   $g_1(n) + g_2(n) \in \Theta(\max(g_1(n), g_2(n)))$

FLIP OVER TO BACK PAGE ⟹

4. **Big-Oh ranking (15 points)**

Rank the following 16 functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

$4^{\log n}$	$\log n$	n^n
2^n	$\log^3 n$	4^{n+4}
4^n	$\sqrt{\log n}$	$\sqrt{\log \sqrt{n}}$
$\log \sqrt{n}$	$n \log n$	n
$\log \log n$	n^2	n^3
$42n + 3n^2$		

As a reminder: $\log^2 n = (\log n)^2$ and $\log \log n = \log(\log n)$. Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{f'(n)}{g'(n)}$$

if the limits exist; where $f'(n)$ and $g'(n)$ are the derivatives of f and g, respectively.