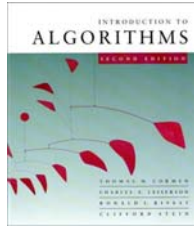




## CS 5633 -- Spring 2009



### Single Source Shortest Paths

Carola Wenk

Slides courtesy of Charles Leiserson with small changes by Carola Wenk

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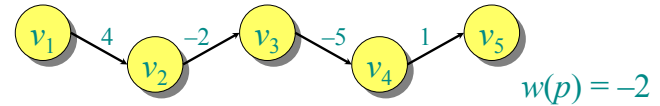


## Paths in graphs

Consider a digraph  $G = (V, E)$  with edge-weight function  $w : E \rightarrow \mathbb{R}$ . The **weight** of path  $p = v_1 \rightarrow v_2 \rightarrow \dots \rightarrow v_k$  is defined to be

$$w(p) = \sum_{i=1}^{k-1} w(v_i, v_{i+1}).$$

**Example:**



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## Shortest paths

A **shortest path** from  $u$  to  $v$  is a path of minimum weight from  $u$  to  $v$ . The **shortest-path weight** from  $u$  to  $v$  is defined as

$$\delta(u, v) = \min \{w(p) : p \text{ is a path from } u \text{ to } v\}.$$

**Note:**  $\delta(u, v) = \infty$  if no path from  $u$  to  $v$  exists.

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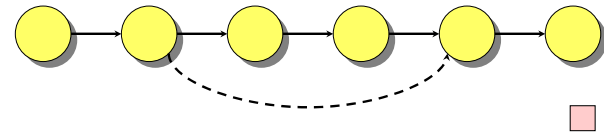
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## Optimal substructure

**Theorem.** A subpath of a shortest path is a shortest path.

**Proof.** Cut and paste:



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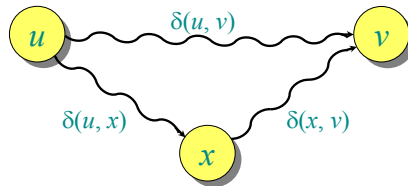


## Triangle inequality

**Theorem.** For all  $u, v, x \in V$ , we have  $\delta(u, v) \leq \delta(u, x) + \delta(x, v)$ .

*Proof.*

- $\delta(u, v)$  minimizes over **all** paths from  $u$  to  $v$
- Concatenating two shortest paths from  $u$  to  $x$  and from  $x$  to  $v$  yields **one** specific path from  $u$  to  $v$



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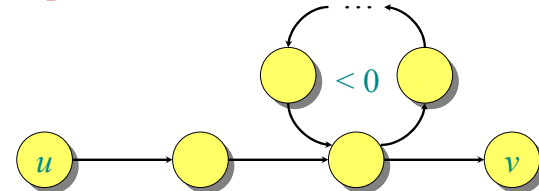
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## Well-definedness of shortest paths

If a graph  $G$  contains a negative-weight cycle, then some shortest paths may not exist.

**Example:**



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## Single-source shortest paths

**Problem.** From a given source vertex  $s \in V$ , find the shortest-path weights  $\delta(s, v)$  for all  $v \in V$ .

If all edge weights  $w(u, v)$  are **nonnegative**, all shortest-path weights must exist.

**IDEA:** Greedy.

1. Maintain a set  $S$  of vertices whose shortest-path weights from  $s$  are known.
2. At each step add to  $S$  the vertex  $v \in V - S$  whose distance estimate from  $s$  is minimal.
3. Update the distance estimates of vertices adjacent to  $v$ .

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## Dijkstra's algorithm

```

d[s] ← 0
for each v ∈ V - {s}
  do d[v] ← ∞
S ← ∅
Q ← V    ▷ Q is a priority queue maintaining V - S
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)

```

*relaxation step*

Implicit DECREASE-KEY

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### Dijkstra

```

PRIM's algorithm
Q ← V
key[v] ← ∞ for all v ∈ V
key[s] ← 0 for some arbitrary s ∈ V
while Q ≠ ∅
  do u ← EXTRACT-MIN(Q)
  for each v ∈ Adj[u]
    do if v ∈ Q and w(u, v) < key[v]
       then key[v] ← w(u, v)
           π[v] ← u

```

$d[s] \leftarrow 0$   
 for each  $v \in V - \{s\}$  do  $d[v] \leftarrow \infty$   
 $S \leftarrow \emptyset$   
 $Q \leftarrow V$   $\triangleright Q$  is  
 while  $Q \neq \emptyset$  do  
    $u \leftarrow \text{EXTRACT-MIN}(Q)$   
    $S \leftarrow S \cup \{u\}$   
   for each  $v \in \text{Adj}[u]$  do  
     if  $d[v] > d[u] + w(u, v)$  then **relaxation step**  
        $d[v] \leftarrow d[u] + w(u, v)$   
       ↑  
       **Implicit DECREASE-KEY**

It suffices to only check  $v \in Q$ , but it doesn't hurt to check all  $v$

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### Example of Dijkstra's algorithm

**Graph with nonnegative edge weights:**

```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)

```

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### Example of Dijkstra's algorithm

**Initialize:**

$S: \{\}$

$Q: \begin{array}{c} A \ B \ C \ D \ E \\ \hline 0 \ \infty \ \infty \ \infty \ \infty \end{array}$

```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)

```

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### Example of Dijkstra's algorithm

**"A" ← EXTRACT-MIN(Q):**

$S: \{A\}$

$Q: \begin{array}{c} A \ B \ C \ D \ E \\ \hline 0 \ \infty \ \infty \ \infty \ \infty \end{array}$

```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)

```

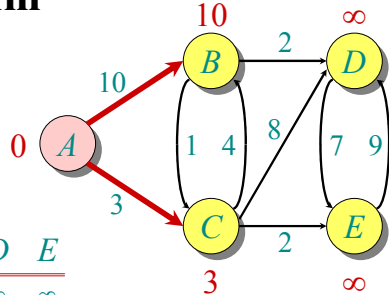
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# Example of Dijkstra's algorithm

Relax all edges leaving A:

S: {A}



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	-	-

```

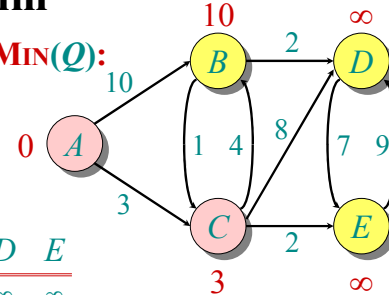
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

"C" ← EXTRACT-MIN(Q):

S: {A, C}



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	-	-

```

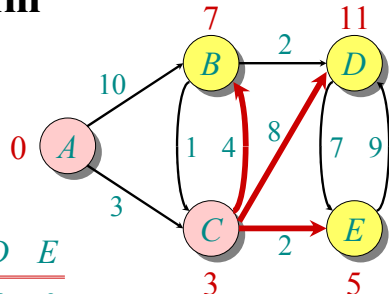
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

Relax all edges leaving C:

S: {A, C}



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	-	-
		7	-	11	5

```

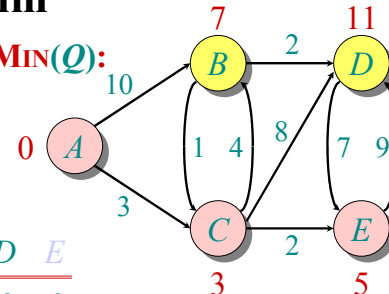
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

"E" ← EXTRACT-MIN(Q):

S: {A, C, E}



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	-	-
		7	-	11	5

```

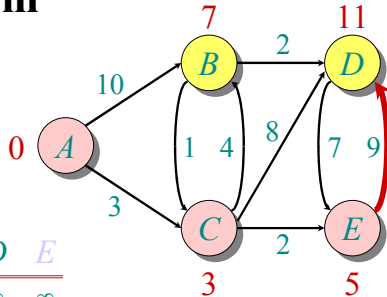
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

Relax all edges leaving **E**:

$S: \{A, C, E\}$



Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11	5	
	7		11		

```

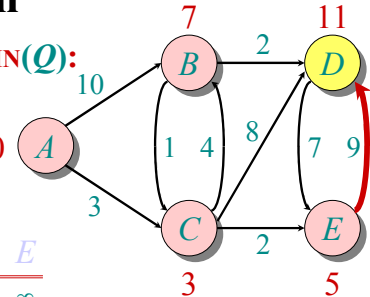
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

**"B"** ← EXTRACT-MIN(Q):

$S: \{A, C, E, B\}$



Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11	5	
	7		11		

```

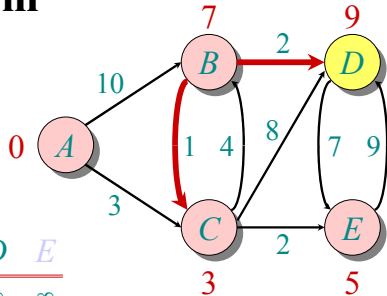
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

Relax all edges leaving **B**:

$S: \{A, C, E, B\}$



Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11	5	
	7		11		

```

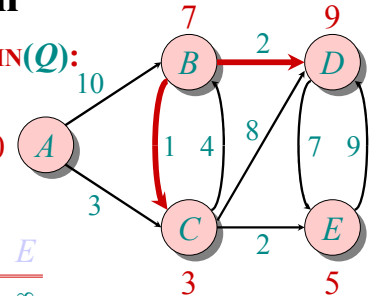
while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



# Example of Dijkstra's algorithm

**"D"** ← EXTRACT-MIN(Q):

$S: \{A, C, E, B, D\}$



Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11	5	
	7		11		

```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
  
```



## Analysis of Dijkstra

$|V|$  times  $\left\{ \begin{array}{l} \text{while } Q \neq \emptyset \text{ do} \\ \quad u \leftarrow \text{EXTRACT-MIN}(Q) \\ \quad S \leftarrow S \cup \{u\} \\ \quad \text{for each } v \in \text{Adj}[u] \text{ do} \\ \quad \quad \text{if } d[v] > d[u] + w(u, v) \text{ then} \\ \quad \quad \quad d[v] \leftarrow d[u] + w(u, v) \end{array} \right.$

$\left. \begin{array}{l} \text{degree}(u) \\ \text{times} \end{array} \right\}$

Handshaking Lemma  $\Rightarrow \Theta(|E|)$  implicit DECREASE-KEY's.

$$\text{Time} = \Theta(|V|) \cdot T_{\text{EXTRACT-MIN}} + \Theta(|E|) \cdot T_{\text{DECREASE-KEY}}$$

**Note:** Same formula as in the analysis of Prim's minimum spanning tree algorithm.

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## Analysis of Dijkstra (continued)

$$\text{Time} = \Theta(|V|) \cdot T_{\text{EXTRACT-MIN}} + \Theta(|E|) \cdot T_{\text{DECREASE-KEY}}$$

$Q$	$T_{\text{EXTRACT-MIN}}$	$T_{\text{DECREASE-KEY}}$	Total
array	$O( V )$	$O(1)$	$O( V ^2)$
binary heap	$O(\log  V )$	$O(\log  V )$	$O( E  \log  V )$
Fibonacci heap	$O(\log  V )$ amortized	$O(1)$ amortized	$O( E  +  V  \log  V )$ worst case

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## Correctness

**Theorem.** (i) For all  $v \in S$ :  $d[v] = \delta(s, v)$   
(ii) For all  $v \notin S$ :  $d[v]$  = weight of shortest path from  $s$  to  $v$  that uses only (besides  $v$  itself) vertices in  $S$ .

**Corollary.** Dijkstra's algorithm terminates with  $d[v] = \delta(s, v)$  for all  $v \in V$ .

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## Correctness

**Theorem.** (i) For all  $v \in S$ :  $d[v] = \delta(s, v)$   
(ii) For all  $v \notin S$ :  $d[v]$  = weight of shortest path from  $s$  to  $v$  that uses only (besides  $v$  itself) vertices in  $S$ .

**Proof.** By induction.

- Base: Before the while loop,  $d[s]=0$  and  $d[v]=\infty$  for all  $v \neq s$ , so (i) and (ii) are true.
- Step: Assume (i) and (ii) are true before an iteration; now we need to show they remain true after another iteration. Let  $u$  be the vertex added to  $S$ , so  $d[u] \leq d[v]$  for all other  $v \notin S$ .

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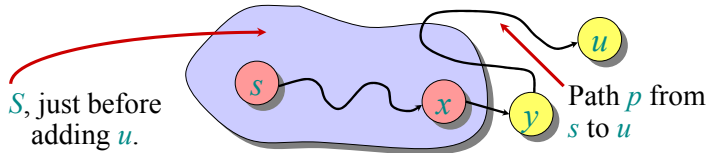
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## Correctness

**Theorem.** (i) For all  $v \in S$ :  $d[v] = \delta(s, v)$   
(ii) For all  $v \notin S$ :  $d[v]$  = weight of shortest path from  $s$  to  $v$  that uses only (besides  $v$  itself) vertices in  $S$ .

- (i) Need to show that  $d[u] = \delta(s, u)$ . Assume the contrary.  
 $\Rightarrow$  There is a path  $p$  from  $s$  to  $u$  with  $w(p) < d[u]$ . Because of (ii) that path uses vertices  $\notin S$ , in addition to  $u$ .  
 $\Rightarrow$  Let  $y$  be first vertex on  $p$  such that  $y \notin S$ .



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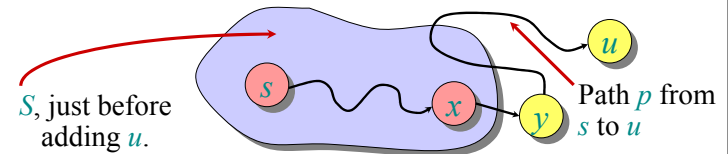
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## Correctness

**Theorem.** (i) For all  $v \in S$ :  $d[v] = \delta(s, v)$   
(ii) For all  $v \notin S$ :  $d[v]$  = weight of shortest path from  $s$  to  $v$  that uses only (besides  $v$  itself) vertices in  $S$ .



$\Rightarrow d[y] \leq w(p) < d[u]$ . Contradiction to the choice of  $u$ .

weights are nonnegative

assumption about path

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## Correctness

**Theorem.** (i) For all  $v \in S$ :  $d[v] = \delta(s, v)$   
(ii) For all  $v \notin S$ :  $d[v]$  = weight of shortest path from  $s$  to  $v$  that uses only (besides  $v$  itself) vertices in  $S$ .

- (ii) Let  $v \notin S$ . Let  $p$  be a shortest path from  $s$  to  $v$  that uses only (besides  $v$  itself) vertices in  $S$ .
  - $p$  does not contain  $u$ : (ii) true by inductive hypothesis
  - $p$  contains  $u$ :  $p$  consists of vertices in  $S \setminus \{u\}$  and ends with an edge from  $u$  to  $v$ .  
 $\Rightarrow w(p) = d[u] + w(u, v)$ , which is the value of  $d[v]$  after adding  $u$ . So (ii) is true.

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## Unweighted graphs

Suppose  $w(u, v) = 1$  for all  $(u, v) \in E$ . Can the code for Dijkstra be improved?

- Use a simple FIFO queue instead of a priority queue.

- **Breadth-first search**

```

while  $Q \neq \emptyset$ 
do  $u \leftarrow \text{DEQUEUE}(Q)$ 
   for each  $v \in \text{Adj}[u]$ 
   do if  $d[v] = \infty$ 
      then  $d[v] \leftarrow d[u] + 1$ 
          ENQUEUE( $Q, v$ )

```

**Analysis:** Time =  $O(|V| + |E|)$ .

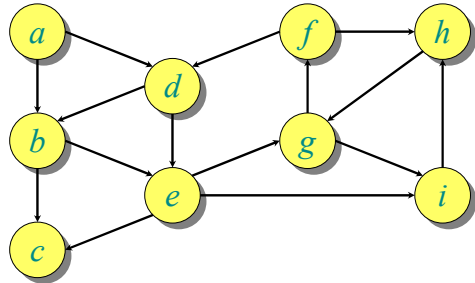
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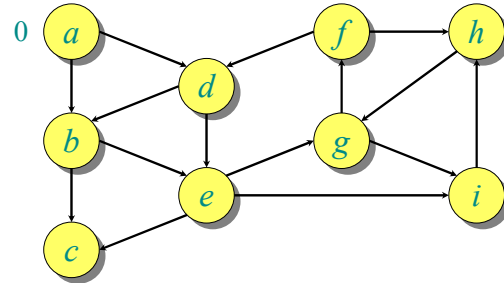
# Example of breadth-first search



Q:



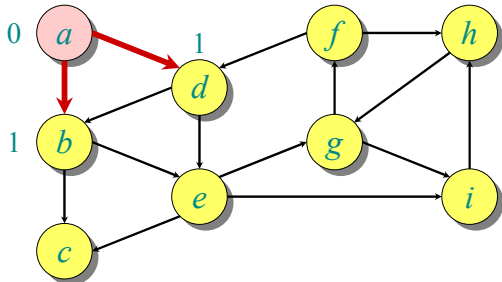
# Example of breadth-first search



Q: a



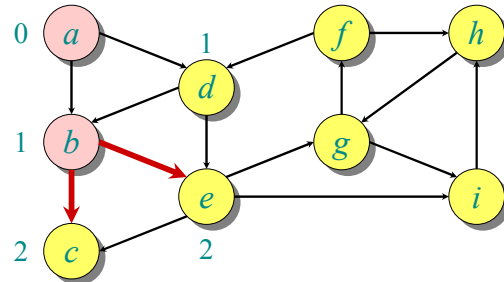
# Example of breadth-first search



Q: a b d



# Example of breadth-first search

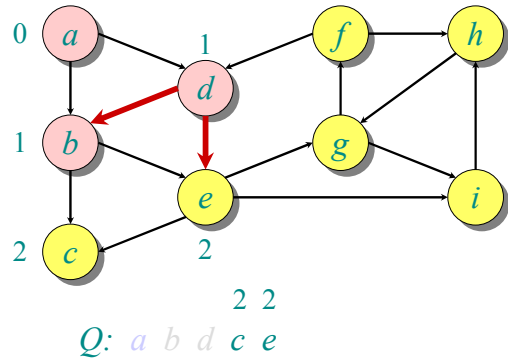


Q: a b d c e

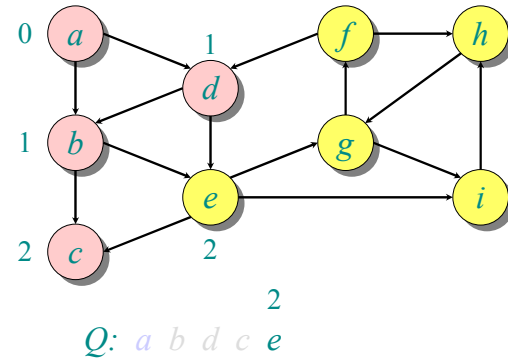




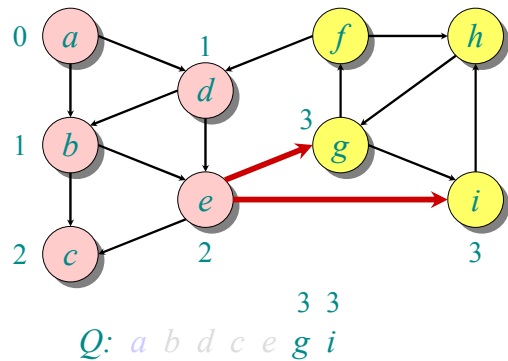
# Example of breadth-first search



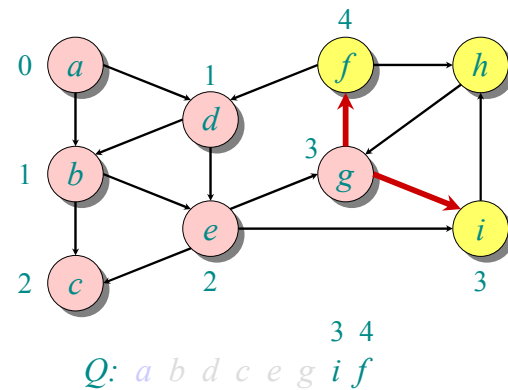
# Example of breadth-first search



# Example of breadth-first search

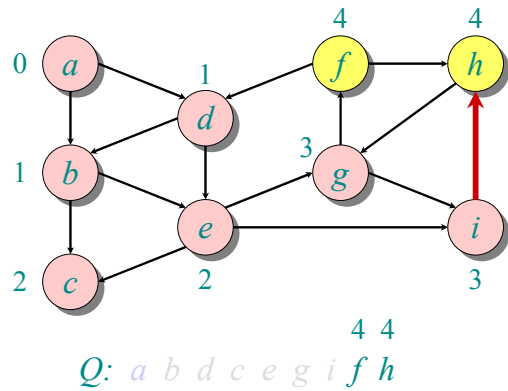


# Example of breadth-first search

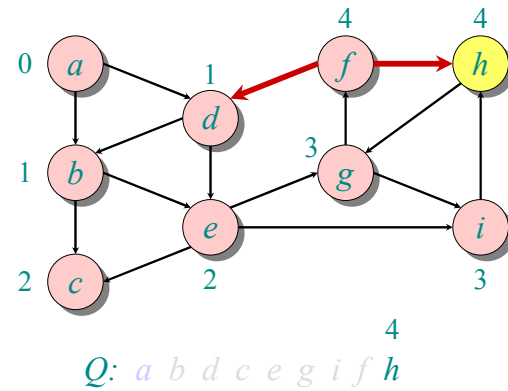




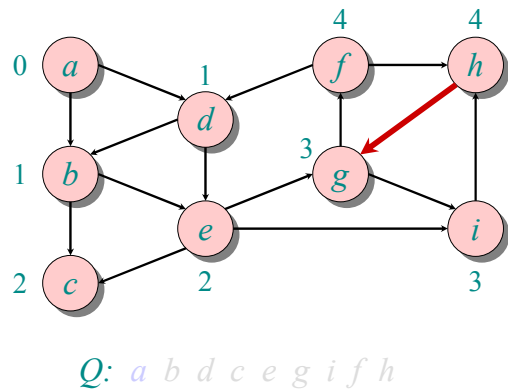
# Example of breadth-first search



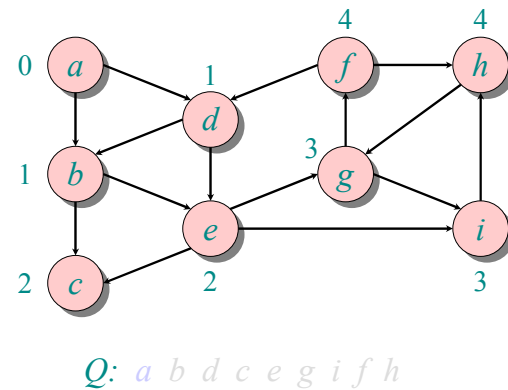
# Example of breadth-first search



# Example of breadth-first search



# Example of breadth-first search





## Correctness of BFS

```

while  $Q \neq \emptyset$ 
  do  $u \leftarrow \text{DEQUEUE}(Q)$ 
    for each  $v \in \text{Adj}[u]$ 
      do if  $d[v] = \infty$ 
          then  $d[v] \leftarrow d[u] + 1$ 
              ENQUEUE( $Q, v$ )

```

### Key idea:

The FIFO  $Q$  in breadth-first search mimics the priority queue  $Q$  in Dijkstra.

- **Invariant:**  $v$  comes after  $u$  in  $Q$  implies that  $d[v] = d[u]$  or  $d[v] = d[u] + 1$ .

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## How to find the actual shortest paths?

### Store a predecessor tree:

```

 $d[s] \leftarrow 0$ 
for each  $v \in V - \{s\}$ 
  do  $d[v] \leftarrow \infty$ 
 $S \leftarrow \emptyset$ 
 $Q \leftarrow V$   $\triangleright Q$  is a priority queue maintaining  $V - S$ 
while  $Q \neq \emptyset$ 
  do  $u \leftarrow \text{EXTRACT-MIN}(Q)$ 
     $S \leftarrow S \cup \{u\}$ 
    for each  $v \in \text{Adj}[u]$ 
      do if  $d[v] > d[u] + w(u, v)$ 
          then  $d[v] \leftarrow d[u] + w(u, v)$ 
               $\pi[v] \leftarrow u$ 

```

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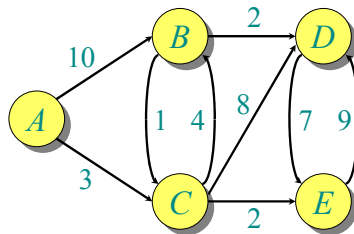
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## Example of Dijkstra's algorithm

Graph with nonnegative edge weights:



```

while  $Q \neq \emptyset$  do
   $u \leftarrow \text{EXTRACT-MIN}(Q)$ 
   $S \leftarrow S \cup \{u\}$ 
  for each  $v \in \text{Adj}[u]$  do
    if  $d[v] > d[u] + w(u, v)$  then
       $d[v] \leftarrow d[u] + w(u, v)$ 
       $\pi[v] \leftarrow u$ 

```

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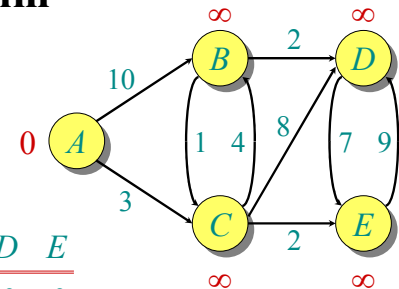


## Example of Dijkstra's algorithm

### Initialize:

$S: \{\}$

$Q: \begin{array}{ccccc} A & B & C & D & E \\ \hline 0 & \infty & \infty & \infty & \infty \end{array}$



```

while  $Q \neq \emptyset$  do
   $u \leftarrow \text{EXTRACT-MIN}(Q)$ 
   $S \leftarrow S \cup \{u\}$ 
  for each  $v \in \text{Adj}[u]$  do
    if  $d[v] > d[u] + w(u, v)$  then
       $d[v] \leftarrow d[u] + w(u, v)$ 
       $\pi[v] \leftarrow u$ 

```

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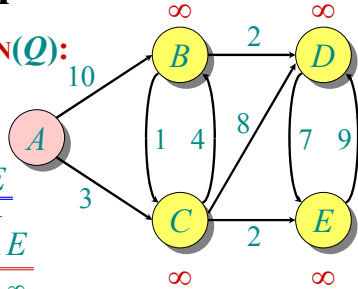
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# Example of Dijkstra's algorithm

"A" ← EXTRACT-MIN(Q):

S:	{ A }	0				
$\pi$ :	A	B	C	D	E	
	-	-	-	-	-	
Q:	A	B	C	D	E	
	0	$\infty$	$\infty$	$\infty$	$\infty$	



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
       $\pi[v] \leftarrow u$ 

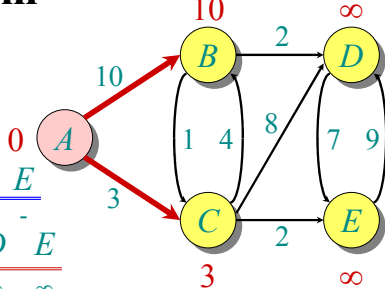
```



# Example of Dijkstra's algorithm

Relax all edges leaving A:

S:	{ A }	0				
$\pi$ :	A	B	C	D	E	
	-	-	-	-	-	
Q:	A	B	C	D	E	
	0	$\infty$	$\infty$	$\infty$	$\infty$	
		10	3	-	-	



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
       $\pi[v] \leftarrow u$ 

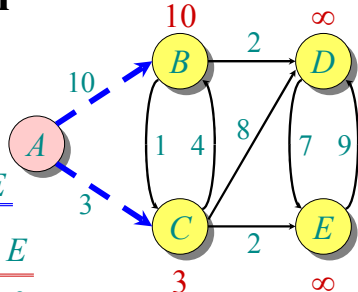
```



# Example of Dijkstra's algorithm

Relax all edges leaving A:

S:	{ A }	0				
$\pi$ :	A	B	C	D	E	
	-	A	A	-	-	
Q:	A	B	C	D	E	
	0	$\infty$	$\infty$	$\infty$	$\infty$	
		10	3	-	-	



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
       $\pi[v] \leftarrow u$ 

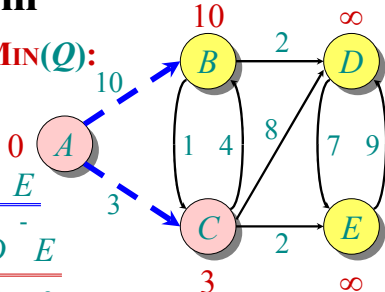
```



# Example of Dijkstra's algorithm

"C" ← EXTRACT-MIN(Q):

S:	{ A, C }	0				
$\pi$ :	A	B	C	D	E	
	-	A	A	-	-	
Q:	A	B	C	D	E	
	0	$\infty$	3	$\infty$	$\infty$	
		10		-	-	



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
       $\pi[v] \leftarrow u$ 

```



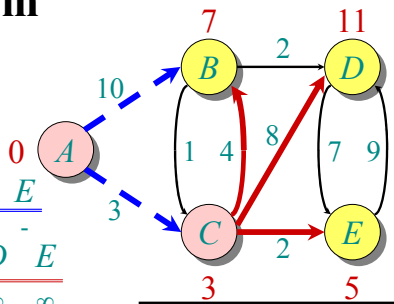
# Example of Dijkstra's algorithm

Relax all edges leaving C:

$S: \{A, C\}$

$\pi: \begin{array}{ccccc} A & B & C & D & E \\ - & A & A & - & - \end{array}$

$Q: \begin{array}{ccccc} A & B & C & D & E \\ \hline 0 & \infty & \infty & \infty & \infty \\ & 10 & 3 & - & - \\ & 7 & & 11 & 5 \end{array}$



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```



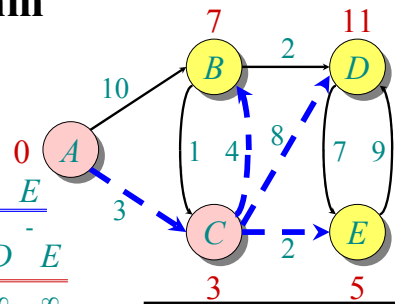
# Example of Dijkstra's algorithm

Relax all edges leaving C:

$S: \{A, C\}$

$\pi: \begin{array}{ccccc} A & B & C & D & E \\ - & A & A & - & - \end{array}$

$Q: \begin{array}{ccccc} A & B & C & D & E \\ \hline 0 & \infty & \infty & \infty & \infty \\ & 10 & 3 & - & - \\ & 7 & & 11 & 5 \end{array}$



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```



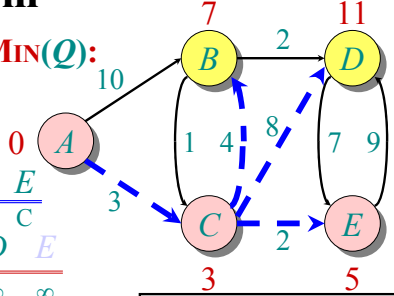
# Example of Dijkstra's algorithm

"E" ← EXTRACT-MIN(Q):

$S: \{A, C, E\}$

$\pi: \begin{array}{ccccc} A & B & C & D & E \\ - & C & A & C & C \end{array}$

$Q: \begin{array}{ccccc} A & B & C & D & E \\ \hline 0 & \infty & \infty & \infty & \infty \\ & 10 & 3 & - & - \\ & 7 & & 11 & 5 \end{array}$



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```



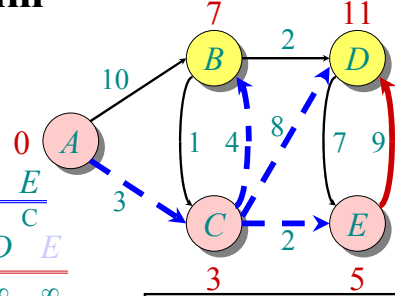
# Example of Dijkstra's algorithm

Relax all edges leaving E:

$S: \{A, C, E\}$

$\pi: \begin{array}{ccccc} A & B & C & D & E \\ - & C & A & C & C \end{array}$

$Q: \begin{array}{ccccc} A & B & C & D & E \\ \hline 0 & \infty & \infty & \infty & \infty \\ & 10 & 3 & \infty & \infty \\ & 7 & & 11 & 5 \\ & 7 & & & 11 \end{array}$



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```



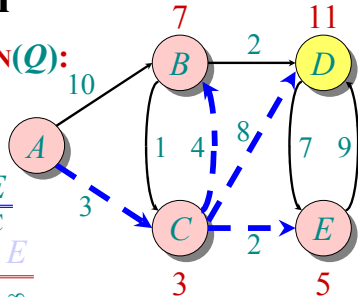
# Example of Dijkstra's algorithm

"B" ← EXTRACT-MIN(Q):

S: {A, C, E, B} 0

$\pi$ :  $\begin{matrix} A & B & C & D & E \\ - & C & A & C & C \end{matrix}$

Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11		5
	7		11		



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```



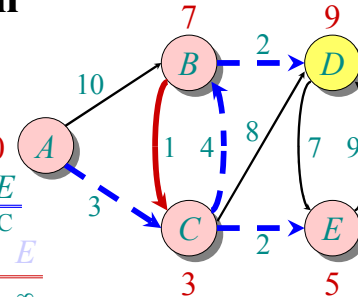
# Example of Dijkstra's algorithm

Relax all edges leaving B:

S: {A, C, E, B} 0

$\pi$ :  $\begin{matrix} A & B & C & D & E \\ - & C & A & B & C \end{matrix}$

Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11		5
	7		11		9



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```



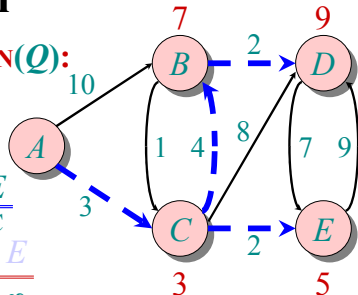
# Example of Dijkstra's algorithm

"D" ← EXTRACT-MIN(Q):

S: {A, C, E, B, D} 0

$\pi$ :  $\begin{matrix} A & B & C & D & E \\ - & C & A & C & C \end{matrix}$

Q:	A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
	10	3	$\infty$	$\infty$	$\infty$
	7		11		5
	7		11		9



```

while Q ≠ ∅ do
  u ← EXTRACT-MIN(Q)
  S ← S ∪ {u}
  for each v ∈ Adj[u] do
    if d[v] > d[u] + w(u, v) then
      d[v] ← d[u] + w(u, v)
      π[v] ← u
  
```