CS 5633 Analysis of Algorithms – Spring 08

3/31/09

8. Homework

Due **4/14/09** before class

1. **Union-Find (4 points)**

```
for(i=1; i<=15; i++) x[i]=MAKE-SET(i);
for(i=2; i<=10; i+=3) UNION(x[i], x[i+1]);
for(i=1; i<=10; i+=3) UNION(x[i], x[i+4]);
UNION(x[1],x[7]); UNION(x[10],x[8]); UNION(x[10],x[2]);
UNION(x[11],x[9]);
FIND-SET(x[14]); FIND-SET(x[15]);
```

   Assume an implementation of the Union-Find data structure with a disjoint-set forest with union-by-weight and path compression.

   Show the data structure after every line of code. What are the answers to the `FIND-SET` operations?

2. **Ackermann (1 points)**
   What is the value of $\alpha(10^{1000})$? Justify your answer.

3. **Adjacency Matrix (6 points)**
   Suppose the graph $G = (V, E)$ is given in an adjacency matrix. The edge weights are the entries in the matrix.

   (a) How fast does BFS run on this graph? Justify your answer.

   (b) How fast does Prim's algorithm run on this graph? Justify your answer.

4. **Prim and Traversals (4 points)**
   Prove whether the following statements are always, never, or sometimes true,

   (a) The order in which Prim's algorithm adds the vertices to the MST is the same as the order in which the vertices are encountered in a breadth-first traversal.

   (b) The order in which Prim's algorithm adds the vertices to the MST is the same as the order in which the vertices are encountered in a depth-first traversal.

5. **Minimum Edge (3 points)**
   Let e be an edge with minimum weight, and assume that there is no other edge with the same weight. Prove that e has to be contained in any minimum spanning tree. (Do NOT use the correctness of Kruskal's algorithm as an argument.)

6. **Finding a Cycle (4 points)**
   Give an algorithm that decides whether an undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in linear time, and with the right arguments you should be able to make it run in $O(|V|)$ time only.