CS 5633 Analysis of Algorithms – Spring 09

1/15/09

1. Homework

Due **1/22/09** before class

1. **Code snippets (4 points)**

   For each of the two code snippets below give their Θ-runtime depending on n. Justify your answers.

   (a) **(3 points)**

   ```
   for(i=n; i>=1; i=i-3){
     for(j=n; j>=1; j=j/3){
       for(k=3*n; k>=1; k--){
         print(" ");
       }
     }
   }
   ```

   (b) **(1 point)**

   ```
   for(i=2; i<=n; i=i*i){
     print(" ");
   }
   ```

2. **O and Ω (4 points)**
   Prove the following, using the definitions of O and Ω:

   - (2 points) $5n^3 + 3n + 2 \in O(n^3)$
   - (2 points) $5n^3 + 3n + 2 \notin \Omega(n^4)$

3. **Selection sort (7 points)**
   Consider sorting n numbers stored in array A by first finding the smallest element of A and exchanging it with the element in $A[1]$. Then find the second smallest element of A, and exchange it with $A[2]$. Continue in this manner for the first $n-1$ elements of A. This algorithm is known as *selection sort*.

   - (2 points) Write pseudocode for this algorithm.
   - (2 points) What loop invariant does this algorithm maintain? Argue (informally) why this loop invariant will help prove the correctness of the algorithm.
   - (1 point) Why does the algorithm need to run for only the first $n-1$ elements, rather than for all n elements?
   - (2 points) Give best-case and worst-case running times (and example inputs attaining these runtimes) of selection sort in Θ-notation.

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4. **Big-Oh ranking (14 points)**
Rank the following functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

$$3n^3 + 4n^4,\ n\log^2 n,\ n^3,\ \log\log n,\ 2^n,\ \log^2 n,\ \sqrt{n},\ \sqrt[3]{n},\ \log n,\ n^n,\ n,\ n\log n,$$
$$2^{n+2},\ 4^n,\ \log\sqrt{n}$$

As a reminder: $\log^2 n = (\log n)^2$ and $\log\log n = \log(\log n)$. Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

if the limits exist; where $f'(n)$ and $g'(n)$ are the derivatives of f and g, respectively.