

10. Homework

Due 4/17/08 before class

1. Negative edge weights (3+2 points)

- Give an example of a directed connected graph with real edge weights (that may be negative) for which Dijkstra's algorithm produces incorrect answers. Justify your answer.
- Does Dijkstra's algorithm only produce incorrect answers in the presence of a negative weight cycle, or could it also produce incorrect answers in the mere presence of negative weight edges (without any negative weight cycles)? Justify your answer.

2. Dijkstra and MSTs (3 points)

Give an example of an undirected weighted graph G in which Dijkstra's shortest path tree (with source s) and a minimum spanning tree are different. (Notice that an undirected edge can be represented as two directed edges, so an undirected graph can be considered as a directed graph as well)

3. Negative-weight cycle (5 points)

Given a directed weighted connected graph $G = (V, E)$ with **real** edge weights (i.e., **negative** edge weights are allowed). Give an algorithm (in words is enough, but if you need to you can write pseudo-code) that detects **AND prints** out a negative-weight cycle if G contains a negative-weight cycle. What is the runtime of your algorithm?

4. Multiple shortest paths (3+4 points)

In the presence of non-unique edge weights the shortest paths between two vertices do not have to be uniquely determined.

- For every vertex v in the graph in Figure 24.2 on page 585 in the book, show all the different $s - v$ shortest paths.
- Given a directed graph $G = (V, E)$ with weights on the edges, and two vertices $s, t \in V$. The weights may be positive or negative, but every cycle in the graph has strictly positive cost. Give an efficient algorithm that computes the number of shortest $s - t$ paths in G . (The algorithm should not list all the paths, just the number suffices)

5. Floyd-Warshall in less space (4 points)

Show how Floyd-Warshall's algorithm can be implemented to use only $\Theta(n^2)$ space (see problem 25.2-4 on page 634 in the book).

6. Extra credit: Johnson's example (5 points)

This question is for extra credit only and is not mandatory.

Question 25.3-1 on page 640 in the book.