CS 5633 Analysis of Algorithms – Spring 08

1/17/08

1. Homework

Due **1/24/08** before class

1. **Code snippets (6 points)**

   For each of the two code snippets below give their Θ-runtime depending on n. Justify your answers. *(Hint: Analyze one loop at a time, and put it together in the end.)*

   (a) **(3 points)**

   ```
   for(i=n; i>=1; i=i/3){
     for(j=1; j<=n; j=j*3){
       for(k=1; k<=n; k=k*2){
         print(" ");
       }
     }
   }
   ```

   (b) **(3 points)**

   ```
   for(i=n; i>=1; i=i-2){
     print(" ");
   }

   for(i=n; i>=1; i=i-1){
     for(j=n; j>=1; j=j-i){
       print(" ");
     }
   }
   ```

2. Θ **(3 points)**
   Prove using the definition of Θ that $n^3 - 2n^2 + 3n - 5 \in \Theta(n^3)$.

3. **Transitivity (4 points)**
   Show using the definitions of big-Oh and Θ:

   (a) If $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$ then $f(n) \in O(h(n))$.

   (b) If $f(n) \in \Theta(g(n))$ and $g(n) \in \Theta(h(n))$ then $f(n) \in \Theta(h(n))$.

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4. **Big-Oh ranking (14 points)**
Rank the following functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

n^2, n^3, $\log\log n$, 3^n, $\log^2 n$, $\sqrt{n}$, $n^2\sqrt{n}+42n$, $\log n$, 1, n^n, n, $n\log n$, 3^{n+1}, 4^n, $4^{\log n}$

As a reminder: $\log^2 n = (\log n)^2$ and $\log\log n = \log(\log n)$. Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

if the limits exist; where $f'(n)$ and $g'(n)$ are the derivatives of f and g, respectively.