CS 5633 Analysis of Algorithms – Spring 06

2/23/06

Practice test questions

1 Big-Oh – 6 points

Rank the following functions according to their asymptotic order of growth. Additionally, indicate which functions are in Θ of each other. (Just as on the first homework). You **do not** need to justify your answers.

$$42,\ \log_3 n,\ n^n,\ n+\log n,\ \sqrt{n},\ \log_2 n,\ 1,\ n!,\ n,\ \sqrt[3]{n},\ n\log n,\ n^3$$

2 Master Theorem – 6 points

Solve the following recurrences using the master theorem. Justify your answers shortly.

1. $T(n) = 36T(n/6) + n^3$
2. $T(n) = 7T(n/2) + \sqrt{n}$

3 Recurrence – 6 points

Consider the following recursive procedure:

```
mystery(n)
 if n <= 1 return 1
 else
   x = mystery(n/3)
   return(x * x * x)
```

1. Set up a runtime recurrence for `mystery`(n).
2. What does the runtime recurrence solve to? (No justification needed.)

4 Code Snippets – 8 points

For each of the code snippets below give their big-Oh runtime depending on n. Make your bounds AS TIGHT AS POSSIBLE. (No justification needed.)

a)
```
for(i=n; i>=1; i=i/2){
  for(j=1; j<=n; j=j+5){
    print("hello");
  }
}
```

b)
```
for(i=1; i*i<=n; i=i+1){
  for(j=1; j<=n; j=j*5){
    print("hello");
  }
}
```

5 Sorting Runtimes – 8 points

Consider the input array

$$4, 5, 6, \ldots, n, n+1, n+2, n+3$$

of n numbers.

1. What is the runtime of running **Insertion Sort** on this array? (no justification needed)
2. What is the runtime of running **Mergesort** on this array? (no justification needed)
3. What is the runtime of running **Deterministic Quicksort** (pivot = first array element) on this array? (no justification needed)
4. What is the **expected** runtime of running **Randomized Quicksort** on this array? (no justification needed)

6 Expected Value – 8 points

Consider playing the following game: You roll a fair six-sided die. If a 1 or a 2 comes up you pay 2$, if a 3 or a 4 comes up you pay 1$, and if a 5 or a 6 comes up you get 4$.

1. What is the underlying sample space S?
2. What is the probability of rolling a 3?
3. Define a random variable X that describes the win/loss of the game. (Remember, a random variable is a function of elements in S)
4. Compute the expected value of X.