CS 5633 Analysis of Algorithms – Spring 06

4/6/06

9. Homework

Due **Thursday 4/18/06** before class

1. **Ackermann (2 points)**
   What is the value of $\alpha(10^{200})$? Justify your answer. What is the value of $\log_2 10^{200}$?

2. **Faster MST (4 points)**
   Let $G = (V, E)$ be a connected undirected graph with edge weights $w : E \to \mathbb{R}$.

   If all of the edge weights are integers between 1 and $|E|$, how fast can the minimum spanning tree be computed? (Give the *most efficient* algorithm you can think of.)

3. **Adjacency Matrix (6 points)**
   Suppose the graph $G = (V, E)$ is given in an adjacency matrix. The edge weights are the entries in the matrix.

   (a) How fast does BFS run on this graph? Justify your answer.

   (b) How fast does Kruskal's algorithm run on this graph? Justify your answer.

4. **Maze (4 points)**
   Let $G = (V, E)$ be a connected undirected graph. Give the most efficient algorithm you can think of to compute a path in G that traverses each edge in E exactly once in each direction. Describe how you can use this algorithm to find your way out of a maze if you are given a large supply of breadcrumbs. Analyze the runtime of your algorithm.

5. **Cycles (4 points)**
   Give the most efficient algorithm you can think of to detect whether a given undirected graph $G = (V, E)$ contains a cycle. If the graph contains a cycle then your algorithm should output one. Analyze the runtime of your algorithm.

6. **DFS and BFS (4 points)**
   Let $G = (V, E)$ be a connected undirected graph, and $u \in V$ one fixed vertex. Let T be the depth-first tree computed by DFS, starting at u. Let T' be the breadth-first tree computed by BFS, starting at u. Show that if $T = T'$ then $G = T$. (In other words, if T is both a depth-first and a breadth-first tree rooted at u, then G cannot contain any edges that do not belong to T.)