CS 5633 Analysis of Algorithms – Spring 06

3/7/06

6. Homework

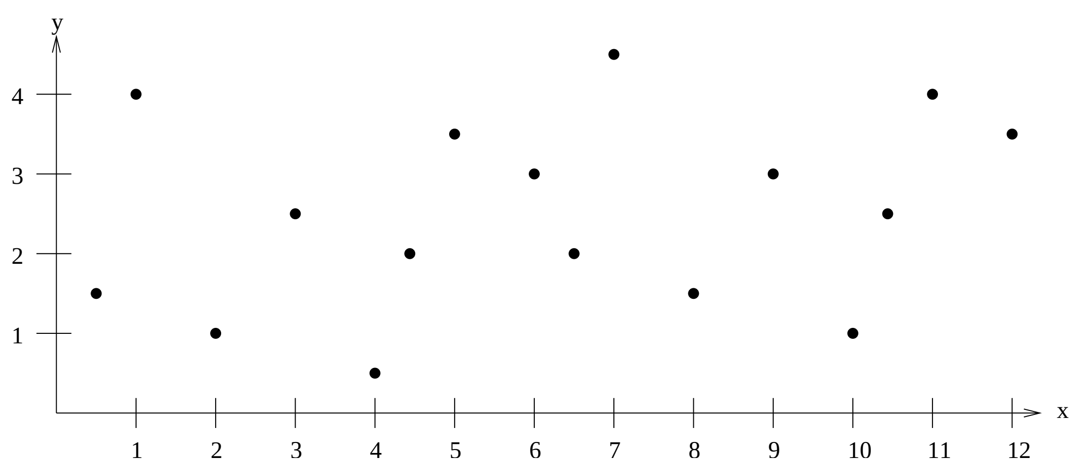
Due **3/23/06** before class

1. **Number of keys in B-trees (4 points)**
   (a) As a function of t and h, what is the minimum number of keys that can be stored in a B-tree with minimum degree t of height h?
   (b) As a function of t and h, what is the maximum number of keys that can be stored in a B-tree with minimum degree t of height h?

2. **Constructing B-trees (9 points)**
   Let $A = \{1, 2, \ldots, 15\}$. When inserting a number into a B-tree please show the different steps that the algorithm performs on the tree as well as the resulting tree.
   Reminder: The height of a single-node tree is 0*, not* 1*.*
   (a) Draw a B-tree with minimum degree 2 and height 1 that contains all numbers of A. Then insert 16 into your B-tree.
   (b) Draw a B-tree with minimum degree 2 and height 2 that contains all numbers of A. Then insert 16 into your B-tree.
   (c) Draw a B-tree with minimum degree 3 and height 1 that contains all numbers of A. Then insert 16 into your B-tree.

3. **Range tree counting queries (4 points)**
   Show how to augment a d-dimensional range tree of n elements such that range counting queries can be answered in $O(log^d n)$ time. Argue that your augmentation does not change the asymptotic preprocessing time and the asymptotic space complexity. *Hint: Start with* $d = 1$*, and then generalize to higher dimensions.*

4. **Range trees (9 points)**
   Let $P = \{(0.5, 1.5), (1, 4), (2, 1), (3, 2.5), (4, 0.5), (4.5, 2), (5, 3.5), (6, 3), (6.5, 2), (7, 4.5), (8, 1.5), (9, 3), (10, 1), (10.5, 2.5), (11, 4), (12, 3.5)\}$ be a set of two-dimensional points.



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a) (3 points) Draw the primary range tree of P. (The keys are the x-coordinates; the leaves store the two-dimensional points, or pointers to them, but not just the x-coordinates).

b) (3 points) Draw all secondary range trees. (The keys are the y-coordinates; the leaves store the two-dimensional points, or pointers to them, but not just the x-coordinates). Notice that since there are duplicate y-coordinates the trees are not unique.

c) (3 points) Consider the query rectangle $[x_1 = 1, x_2 = 6] \times [y_1 = 1.5, y_2 = 4]$. Show how the range reporting query which prints out all points in the query rectangle proceeds in the range tree:

- Show the split nodes (in the primary tree, and in the secondary trees).
- Show the search paths (in the primary tree, and in the secondary trees).
- Show which secondary trees are queried.
- Show which points are output (mark the corresponding leaves in the secondary trees).

Here is another copy of the point set: