CS 5633 Analysis of Algorithms – Spring 06

2/21/06

5. Homework

Due **3/7/06** before class

1. **Red-black trees**
   Find a sequence of numbers which, when incrementally inserted into a red-black tree, causes the following sequence of rotations:

   left, left, right, right.

   You may start with an initially non-empty tree, and you may insert numbers that do not cause any rotations. But there should not be any additional rotations performed.

   Draw the sequence of trees that you obtain after each insertion. For each such tree indicate the node that violates the red-black tree condition, indicate the nodes that participate in the rotation, the type of the rotation, and the subtrees that correspond to each other before and after the rotation.
   Hint: Use a red-black tree demo from the web.

2. **Median computation**
   Suppose arrays A and B are **both sorted** and contain n elements each. Give a randomized divide-and-conquer algorithm to find the median of $A \cup B$ in expected $O(\log n)$ time. (Describe it either in words or as pseudo-code; whatever you prefer). Argue **shortly** why the runtime is $O(\log n)$. *Hint: Take a look at randomized select.*

3. **Multi-min**
   Consider the following task: Given an unsorted array of n numbers, find the k smallest numbers and output them in sorted order.
   Describe three inherently different algorithms that solve this problem. Analyze their runtimes in terms of n and k (so you should have n and k in the big-Oh notation). Try to find the fastest possible algorithm.

4. **Multi-select (6 points)**
   Suppose that we are interested in finding k order statistics, that is, the i_1th largest number, the i_2th largest number, ..., and the i_kth largest number. Revise the randomized SELECT algorithm (call it SELECT-k) so that its average-case running time will be $\theta(n \log k)$ for an array with n elements and the task is to find k order statistics. Provide a persuasive informal argument that your SELECT-k algorithm has $\theta(n \log k)$ average-case running time.