CS 5633 Analysis of Algorithms – Spring 06

1/19/05

1. Homework

Due **1/26/05** before class

1. **Code snippets (6 points)**

   For each of the code snippets below give their Θ-runtime depending on n. Justify your answers.

   (a) **(2 points)**

   ```
   for(i=n; i>=1; i=i/2){
       print("Awesome");
   }
   ```

   (b) **(2 points)**

   ```
   for(i=1; i<n; i=i*2){
     for(j=1; j<=i; j++){
       print("homework");
     }
   }
   ```

   (c) **(2 points)**

   ```
   for(i=1; i*i<=n; i++){
     for(j=i; j>=1; j--){
       print("assignment");
     }
   }
   ```

2. **Polynomial (2 points)**
   Let $\sum_{i=0}^{m} a_i n^i$ be a polynomial in n of degree $m \geq 0$ with coefficients $a_0, \ldots, a_m > 0$. Prove the following, using only the definition of Θ:

   $$\sum_{i=0}^{m} a_i n^i \in \Theta(n^m)$$

3. O, Ω, Θ **(4 points)**

   (a) Use the definition of Θ to prove the following:
   $g_1(n) + g_2(n) \in \Theta(\max(g_1(n), g_2(n)))$

   (b) If $f(n) \in \Theta(g(n))$ and $g(n) \in \Omega(h(n))$. Which of the following is true?
   $f(n) \in O(h(n))$ or $f(n) \in \Omega(h(n))$
   Justify your answer.

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4. **Big-Oh ranking (12 points)**
Rank the following functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

$$n^2,\ n^3,\ \log\log n,\ \log n,\ 1,\ n^n,\ n,\ 2^n,\ \log^2 n,\ n\log n,\ 2^{n+1},\ 3^n,\ \sqrt{n}$$

As a reminder: $\log^2 n = (\log n)^2$ and $\log\log n = \log(\log n)$. Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

if the limits exist; where $f'(n)$ and $g'(n)$ are the derivatives of f and g, respectively.