CS 5633 Analysis of Algorithms – Spring 05

4/14/05

7. Homework

Due **4/26/05** before class

1. **Examples (6 points)**

   Please add, to each of your examples below, a description as to why this example causes the specified problem.

   (a) **Dijkstra does not always work in the presence of negative edge weights:**
   Give an example of a directed connected graph with **real** edge weights (i.e., **negative** edge weights are allowed), such that Dijkstra's algorithm produces incorrect answers.

   (b) **Breadth-first search only works for unweighted graphs:**
   Give an example of a directed graph with **real edge weights**, in which breadth-first search does NOT compute the correct shortest path weights from a single source. Can you do it with nonnegative edge weights, or do you have to use negative edge weights?

2. **Negative-weight cycle (5 points)**
   Given a directed weighted connected graph $G = (V, E)$ with **real** edge weights (i.e., **negative** edge weights are allowed). Give an algorithm (in words is enough, but if you need to you can write pseudo-code) that detects **AND prints** out a negative-weight cycle if G contains a negative-weight cycle. What is the runtime of your algorithm?

3. **Flow lemma (6 points)**
   Prove the lemma on slide 12 of the Flow Networks slides.

4. **Ford-Fulkerson (5 points)**
   Run the Ford-Fulkerson algorithm on the network on the back-side of this page:

   - Start with a zero flow on all edges and initialize the residual network. Then repeat: Find an augmenting path in the residual network (draw it in), write down the value by which you will increase the flow. Then increase the flow in the graph and update the residual network.
   - You do not need to draw in 0-capacity edges into the network, but remember that negative flow on them might have effect on the residual network.
   - Remember the formula for the residual capacities: $c_f(u, v) = c(u, v) - f(u, v)$

5. **To be in NP or not to be in NP (5 points)**
   Which of the problems below are in NP and which are not? Justify your answers.

   (a) Given a directed acyclic graph G. Topologically sort G.

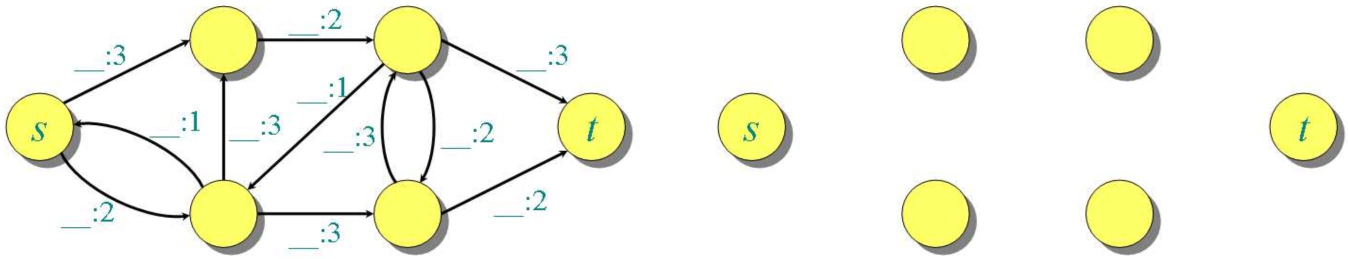
   (b) Given a directed graph G. Is G acyclic?

   (c) Given an unsorted array A of n numbers. What is the fifth smallest element in A?

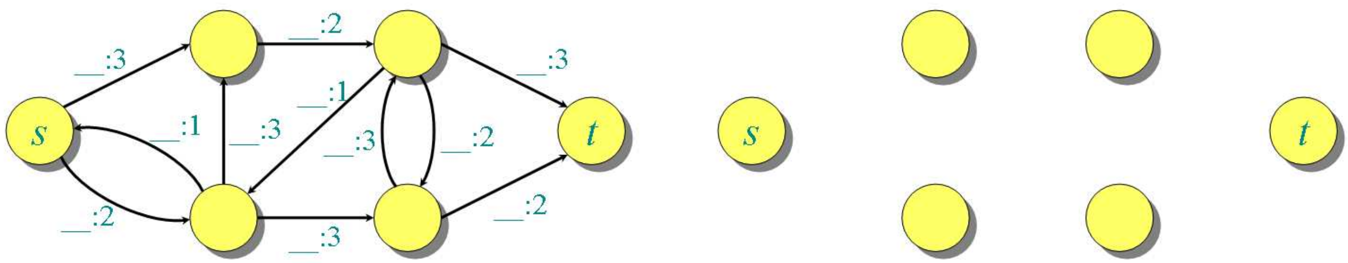
   (d) Given n points in the plane and two parameters $r, \varepsilon > 0$. Is there a path of length r such that the sum of the distances of the points to the path is at most ε? (The distance of a point to a path is the smallest distance from the point to any point on the path.)

   (e) Given a positive integer i. Is i not a prime number (i.e., is it the product of two integers greater than 1)?

Network Residual network



Increase flow value by:



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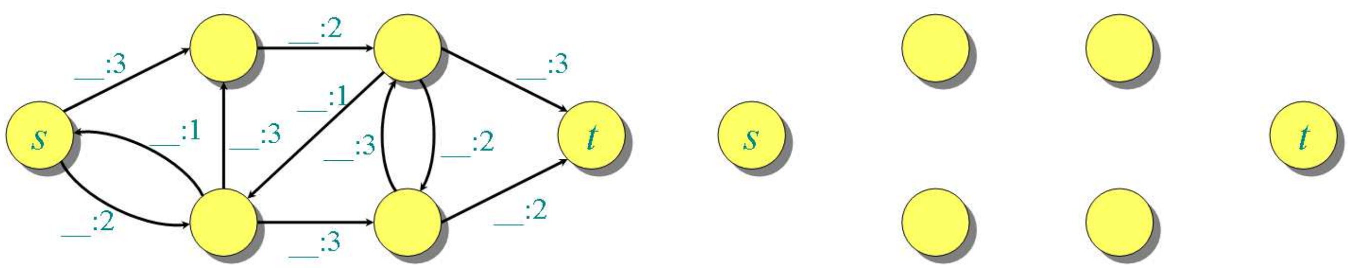

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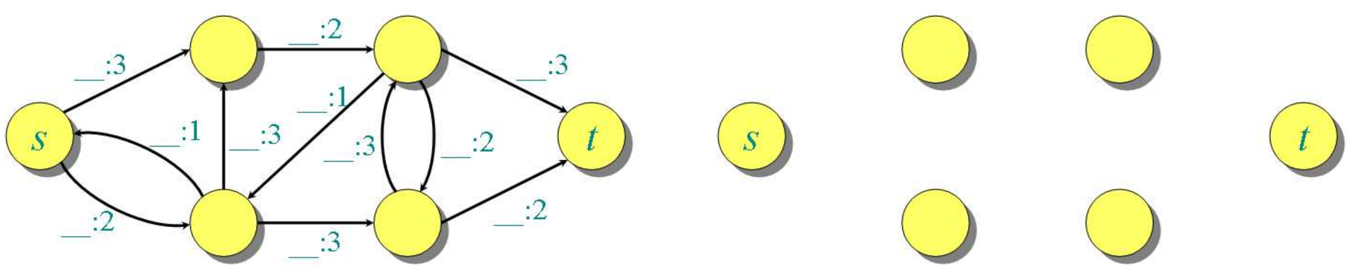

Increase flow value by: