CS 5633 Analysis of Algorithms – Spring 05

3/10/05

5. Homework

Due **3/24/05** before class

1. **B-trees (3 points)**
   Why does the minimum-degree parameter k of a B-tree have to be greater then 1? What would happen if we set $k = 1$?

2. **Saving space (8 points)**

   (a) The bottom-up dynamic programming algorithm computing the n-th Fibonacci number $F(n)$ takes $O(n)$ time and uses $O(n)$ space. Show how to modify the algorithm to use only constant space.

   (b) Suppose we only want to compute the *length* of an LCS of two strings of length m and n. Show how to alter the dynamic programming algorithm such that it only needs $\min(m, n) + O(1)$ space. (Notice that it is *not* $O(\min(m, n))$, but plain $\min(m, n)$.) *(Hint: First, try to make it run in $m + O(1)$ or in $n + O(1)$ space.)*

3. **Presents (14 points)**
   Carola is going home to Germany for a visit. Her parents and friends bought her a lot of presents, amongst them books and other heavy objects. Unfortunately, the airline only allows her baggage to weigh at most 140 pounds. (Assume that the size of the presents does not matter.)

   Assume there are n items: item i has value v_i and weight w_i. Help Carola to select a set of items which maximizes the total value while not exceeding a total weight of $W = 140$ pounds. Proceed in the following steps:

   (a) Come up with an example which shows that the **greedy** solution (pick presents $1, 2, 3, \ldots$ until the weight limit is met) does not produce the optimal solution.

   (b) Since **greedy** does not work, design a **dynamic programming** algorithm:

   - Let $D[i, w]$ be the value of a solution considering presents $1..i$ only and with maximum weight w. Come up with a recurrence relation for $D[i, w]$. *(Hint: it has two non-trivial cases, and it should use the maximum at some point.)*
   - Use dynamic programming to compute $D[n, W]$. What is the runtime in terms of n and W?
   - Extract the optimum set of presents from the dynamic programming table. What is the runtime in terms of n and W?