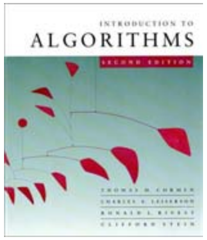
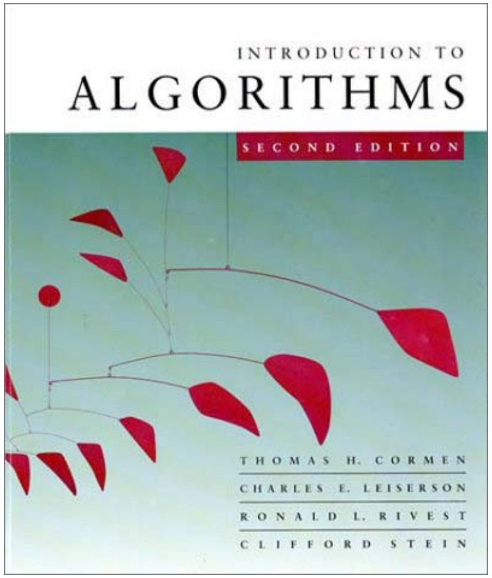


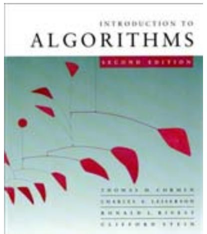
CS 5633 -- Spring 2004



Quicksort (correction)

Carola Wenk

Slides courtesy of Charles Leiserson with small changes by Carola Wenk



Hairy recurrence

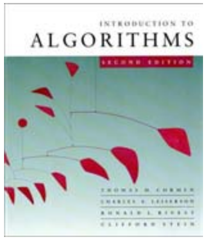
$$E[T(n)] = \frac{2}{n}\sum_{k=2}^{n-1} E[T(k)] + \Theta(n)$$

(The $k = 0, 1$ terms can be absorbed in the $\Theta(n)$.)

Prove: $E[T(n)] \le a\,n\log n$ for constant $a > 0$.

- Choose a large enough so that $a\,n\log n$ dominates $E[T(n)]$ for sufficiently small $n \ge 2$.

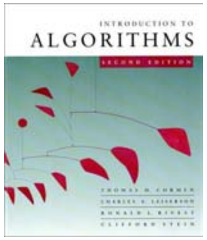
Use fact: $\sum_{k=2}^{n-1} k\log k \le \frac{1}{2}n^2\log n - \frac{1}{8}n^2$ (exercise).



Substitution method

$$E[T(n)] \le \frac{2}{n} \sum_{k=2}^{n-1} ak \log k + \Theta(n)$$

Substitute inductive hypothesis.

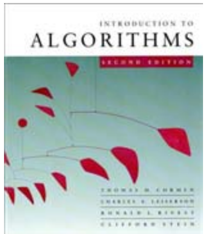


Substitution method

$$E[T(n)] \le \frac{2}{n}\sum_{k=2}^{n-1} ak\log k + \Theta(n)$$

$$\le \frac{2a}{n}\left(\frac{1}{2}n^2\log n - \frac{1}{8}n^2\right) + \Theta(n)$$

Use fact.



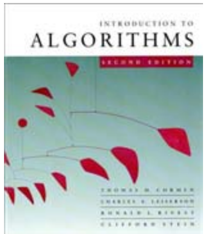
Substitution method

$$E[T(n)] \le \frac{2}{n}\sum_{k=2}^{n-1} ak\log k + \Theta(n)$$

$$\le \frac{2a}{n}\left(\frac{1}{2}n^2\log n - \frac{1}{8}n^2\right) + \Theta(n)$$

$$= an\log n - \left(\frac{an}{4} - \Theta(n)\right)$$

Express as ***desired – residual***.



Substitution method

$$E[T(n)] \le \frac{2}{n}\sum_{k=2}^{n-1} ak\log k + \Theta(n)$$

$$= \frac{2a}{n}\left(\frac{1}{2}n^2\log n - \frac{1}{8}n^2\right) + \Theta(n)$$

$$= an\log n - \left(\frac{an}{4} - \Theta(n)\right)$$

$$\le an\log n$$

,

if a is chosen large enough so that $an/4$ dominates the $\Theta(n)$.