4/21/04

Extra Credit Homework

Due **4/26/04** before class

This homework is NOT mandatory.

1. **Transitivity (4 points)**

   Show the transitivity property of the polynomial-time reduction " $\leq''$ (fact 3 on slide 14):
   Let Π, Π', Π'' be three problems. If $\Pi \leq \Pi'$ and $\Pi' \leq \Pi''$ then $\Pi \leq \Pi''$.

2. **To be in NP or to not be in NP (5 points)**
   Which of the problems below are in NP, and which are not? Justify your answers.

   a) Given a positive integer a, is a not a prime number?

   b) Given a positive integer b which is known not to be a prime number, factorize it (i.e., find two integers > 1 whose product equals b).

   c) Given two undirected graphs G_1, G_2, are they isomorphic? (Two graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ are called *isomorphic* if there exists a 1-to-1 map $f : V_1 \to V_2$ such that $(u, v) \in E_1$ iff $(f(u), f(v)) \in E_2$.

   d) Given an undirected graph G. Is G connected?

   e) Compute an MST of an undirected graph with edge weights.

3. **Subgraph isomorphism (5 points)**
   Problem 34.5-1 on page 1017.
   Hint: Show that the problem is in NP, and then show that it is NP-hard. For the NP-hardness you need to pick an NP-hard problem, and polynomially reduce it to the subgraph-isomorphism problem.

4. **Consequences (8 points)**

   a) If somebody finds a polynomial time algorithm A to solve the NP-complete problem Π in polynomial time. What consequences does this have for the problem CLIQUE?

   b) If somebody finds a polynomial time algorithm A to solve the problem Π, which is known to lie in NP, in polynomial time. What consequences does this have for the problem CLIQUE?

   c) If somebody finds an exponential time algorithm A to solve the NP-complete problem Π in polynomial time. What consequences does this have for the problem CLIQUE?

   d) If somebody shows that any algorithm A which solves the NP-complete problem Π, needs at least exponential time. What consequences does this have for the problem CLIQUE?

5. **P=NP? (3 points)**
   Which of the following statements are correct? Justify your answers.
   a) P$\subseteq$ NP b) NP$\subseteq$ P c) P=NP