CS 5633 Analysis of Algorithms – Spring 04

2/11/04

5. Homework

Due **2/18/04** before class

As usual, justify all your answers.

1. **Sorting algorithms (6 points)**
   Consider the four sorting algorithms **mergesort**, **quicksort**, **heapsort**, **counting sort**.

   a) Which of the three algorithms are *stable*?

   b) Which of the three algorithms sort *in-place*? (An algorithm sorts *in-place* if it needs only a constant amount of extra space.)

   c) Do you think that it would be possible to slightly modify the algorithms to make them work in-place or to make them stable?

2. **Decision tree for merge sort (3 points)**
   Draw the decision tree for merge sort for inputs of length 3.

3. **Leafs in decision trees (4 points)**
   Consider a decision tree for a comparison sort of n elements.

   a) How close to the root can a leaf be, i.e., what is the smallest possible depth (the highestmost possible layer) at which a leaf can be?

   b) How far away from the root can a leaf be, i.e., what is the largest possible depth (the lowestmost possible layer) at which a leaf can be?

4. **Median computation (7 points)**
   Suppose arrays A and B are **both sorted** and both contain n elements. Give a randomized divide-and-conquer algorithm to find the median of $A \cup B$ in expected $O(\log n)$ time. (Describe it either in words or as pseudo-code; whatever you prefer). Argue **shortly** why the runtime is $O(\log n)$. *Hint: Take a look at randomized select.*

4. **Radix (1 point)**
   How many digits are there when a 64-bit quantity is viewed as a radix-128 number? Describe how to extract each of the digits.

5. **Ranges (7 points)**
   Given n integers each between 0 and k (inclusive). Give an algorithm which preprocesses the input in $O(n + k)$ time such that the following query can be answered in $O(1)$ time: "How many of the integers are in the range $[a, b]$ (where a, b are query arguments) ?" *Hint: Take a look at counting sort.*

6. **Sort in linear time (2 points)**
   Given n integers in the range 0 to $n^4 - 1$. How fast can you sort them?