CS 5633 Analysis of Algorithms – Spring 04

1/28/04

3. Homework

Due **2/4/04** before class

From now on, if you are asked to design an algorithm for a problem, it always means that you should be as efficient as possible, and analyze the runtime. Also, always justify your answers.

1. **Master theorem (10 points)**
   Use the master theorem to find tight asymptotic bounds for the following recurrences. Justify your results.

   Assume that $T(n)$ is constant for $n \leq 2$.

   - **(2 points)**
     $T(n) = 4T(\frac{n}{3}) + n^4$
   - **(2 points)**
     $T(n) = T(\frac{n}{2}) + \sqrt{n}$
   - **(2 points)**
     $T(n) = T(\frac{7n}{8}) + n$
   - **(2 points)**
     $T(n) = 9T(\frac{n}{3}) + n^2$
   - **(2 points)**
     $T(n) = 5T(\frac{n}{2}) + n^2$

2. **Polynomial multiplication (10 points)**

   - **a) (3 points)**
     Show how to multiply two degree-1 polynomials $ax + b$ and $cx + d$ using only three multiplications. *Hint: One of the multiplications is* $(a + b) \cdot (c + d)$.
   - **b) (7 points)**
     Design a divide&conquer algorithm for multiplying two polynomials of degree n (so, of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 n + a_0$) in time $\Theta(n^{\log_2 3})$. *Hint: Split the input polynomials into two parts, based on the parity of the powers of* x. *Then use part* **a)** *with polynomials of degree* $n/2$ *(in* x^2*) instead of constants.*

3. **Rolling dice (5 points)**

   - **a) (3 points)**
     Use indicator variables to compute the expected value of the sum of two 10-sided dice.
   - **b) (2 points)**
     Use indicator variables to compute the expected value of the sum of k n-sided dice. The sum should be expressed in terms of k and n.

   Clearly describe the sample space and the random variables you use.

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4. **Roulette (3 points)**

   The game SimpleRoulette is played as follows: The roulette wheel has a slot for each number from 0 to 36. You can bet on any number between 1 and 36, but not on the number 0. A bet costs you \$10. If the ball drops on the slot with your number, you get paid \$360, otherwise you don't get paid anything.

   Assuming that the wheel is fair (i.e., all numbers are equally likely), what is your expected win/loss in this game?

   Clearly describe the sample space and the random variables you use.

5. **Reading (2 points)**
   Read sections 5.1, 5.2, and 5.3. In order to receive the two points, write something to convince me that you actually read it.