4/14/04

11. Homework

Due **4/21/04** before class

This is the last mandatory homework !!!

1. **Negative edge weights (2+2 points)**

   a) Give a counterexample of a directed connected graph with negative edge weights for which Dijkstra's algorithm produces incorrect answers. (Describe where the problem is, and don't just draw the example.)

   b) In the case of negative edge weights, why doesn't the proof of the correctness of Dijkstra's algorithm go through? Where in the proof did we assume that the edge weights are nonnegative?

2. **Dijkstra variations (4 points)**
   Explain what adjustments, if any, need to be made in Dijkstra's algorithm and/or in an underlying graph to solve the following problems efficiently. What are the runtimes of your algorithms?

   a) Solve the single-source shortest-paths problem for undirected weighted graphs.

   b) **Single-pair shortest-path:** Find a shortest path between two given vertices of a weighted digraph.

   c) **Single-destination shortest-paths:** Find a shortest path to a given vertex from each other vertex of a weighted digraph.

   d) Solve the single-source shortest-path problem in a (directed or undirected) graph with nonnegative weights assigned to its vertices, and define the length of a path to be the sum of the vertex weights comprising the path.

3. **Shortest paths trees (2 points)**
   Give two shortest paths trees for the graph in Fig 24.2 on page 585, which are different from the ones shown in the figure.

4. **Dijkstra and MSTs (2+2 points)**
   Let T be the tree constructed by Dijkstra's algorithm for a weighted digraph G. Prove or disprove:

   a) T is a spanning tree of G.

   b) T is a minimum spanning tree of G.

5. **All pairs shortest paths in a DAG (4 points)**
   Given a directed acyclic graph (a DAG) with real edge weights. How fast can you compute the shortest-path weights for all pairs of vertices? (Of course, justify your answer by giving your algorithm!)

6. **Floyd-Warshall in less space (3 points)**
   Problem 25.2-4 on page 634 in the book.

7. **Floyd-Warshall and negative-weight cycles (3 points)**
   Problem 25.2-6 on page 635 in the book.