CS 5633 Analysis of Algorithms – Spring 04

4/7/04

10. Homework

Due **4/14/04** before class

1. **Union-Find (8 points)**

```
for i:=1 to 16 do MAKE-SET(x[i])
for i:=2 to 14 by 2 do UNION(x[i], x[i+1])
for i:=1 to 11 by 5 do UNION(x[i], x[i+3])
UNION(x[1],x[5])
UNION(x[11],x[13])
UNION(x[1],x[9])
FIND-SET(x[2])
FIND-SET(x[8])
```

   a) Assume an implemention of the Union-Find data structure with the (augmented) linked-listed representation with the weighted-union heuristic. Assume that if the sets containing `x[i]` and `x[j]` have the same size, then the operation `UNION(x[i],x[j])` appends `x[j]`'s list onto `x[i]`'s list.

   Show the data structure after every for-loop, as well as the final data structure. What are the answers to the `FIND-SET` operations?

   b) Same as in part a), but this time using a disjoint-set forest with union-by-weight and path compression.

2. **Lower-bound example (5 points)**

   Give a sequence of m `MAKE-SET`, `UNION`, and `FIND-SET` operations on elements $x_1, x_2, \ldots)$, where n of the operations are `MAKE-SET` operations, such that the whole sequence takes $\Omega(m \log n)$ time when using a disjoint-set forest with union-by-weight only.

3. **Unique MST (4 points)**
   Let $G = (V, E)$ be a connected undirected graph with edge weights $w : E \to \mathbb{R}$.

   Show that if the edge weights are all different, then there exists a unique MST for G.

   Hint: Consider the sorted sequence of edge weights. Use an indirect proof (a proof "by contradiction").

4. **Faster MST (4 points)**
   Let $G = (V, E)$ be a connected undirected graph with edge weights $w : E \to \mathbb{R}$.

   If all of the edge weights are integers between 1 and $|E|$, how fast can the minimum spanning tree be computed? (Give the *most efficient* algorithm you can think of.)

5. **Kruskal's tree (3 points)**
   Argue why Kruskal's algorithm computes a tree. *(Hint: A tree is a connected acyclic graph.)* Do NOT use the fact that somebody else has proven that Kruskal's algorithm computes an MST.

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6. **Ackermann (6 points)**

a) (4 points) Justify why the values of $A_0(1), A_1(1), A_2(1), A_3(1)$ are given as on slide 22 of the Union/Find slides. Give the computation.

b) (2 points) What is $\alpha(10^9)$? Justify your answer.