CS 5633 Analysis of Algorithms – Spring 04

1/14/04

1. Homework

Due **1/21/04** before class

1. **Palindrome checker (7 points)**
   A palindrome is a string whose first half equals its last half reversed. So for example "4456544" or "hannah" or "abcdcba" are palindromes.

   Write an algorithm which checks if a given string is a palindrome, and analyze its runtime. Please give a formal problem description (what are the input and output?), an informal and a formal desription of the algorithm, a proof of correctness (using a loop invariant), and its runtime analysis.

2. **Code snippets (6 points)**

   For each of the code snippets below give their big-Oh runtime depending on n. Make your bound as tight as possible. Justify your answers.

   (a) **(2 points)**

   ```
   for(j=1; j<=n; j=j*7){
     for(i=n; i>=1; i=i/2){
       print(``hello'');
     }
   }
   ```

   (b) **(2 points)**

   ```
   for(j=5; j<=n; j=j*j){
     print(``hello'');
   }
   ```

   (c) **(2 points)**

   ```
   for(i=1; i<=n*n; i+=1){
     for(j=1; j<=i; j++){
       print(``no'');
     }
   }
   ```

3. **Theta (2 points)**
   Prove the following, using only the definitions of O and Θ:
   $f(n) \in \Theta(g(n))$ if and only if $f(n) \in O(g(n))$ and $g(n) \in O(f(n))$

4. **Little-oh (2 points)**
   Is $n + 13n^2 \in o(n^2)$ true? Justify your answer.

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5. **Big-Oh ranking (13 points)**
Rank the following functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f = \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same clase, prove that $f = \Theta(g)$.

$$n^2,\ n!,\ n^3,\ 2^{2^n},\ \log\log n,\ \log n,\ 1,\ 4^{\log n},\ n,\ 2^n,\ n\log n,\ 2^{n+1},\ (3/2)^n,\ \sqrt{n}$$

Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

if the limits exist; where $f'(n)$ and $g'(n)$ are the derivatives of f and g, respectively.