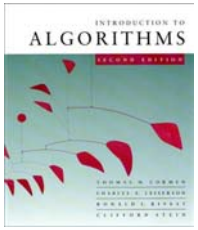
CS 3343 -- Spring 2009



Merge Sort

Carola Wenk

Slides courtesy of Charles Leiserson with small changes by Carola Wenk

Merge sort

MERGE-SORT $A[1 \ldots n]$

1. If $n = 1$, done.
2. Recursively sort $A[\,1 \ldots \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \ldots n\,]$.
3. "***Merge***" the 2 sorted lists.

Key subroutine: **MERGE**

Merging two sorted arrays

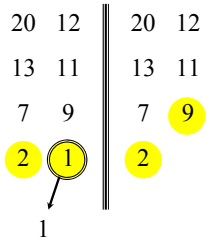
20 12
13 11
7 9
2 1

Merging two sorted arrays

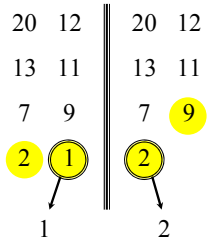
20 12
13 11
7 9
2 1

1

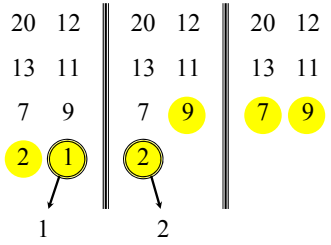
Merging two sorted arrays



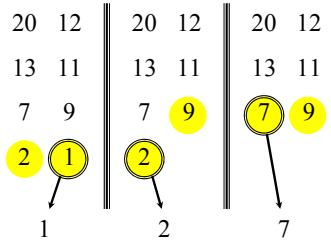
Merging two sorted arrays



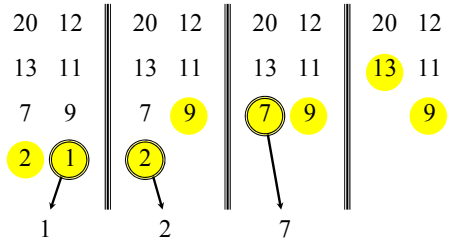
Merging two sorted arrays



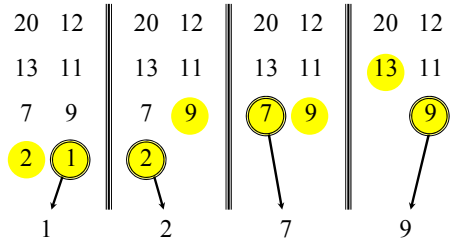
Merging two sorted arrays



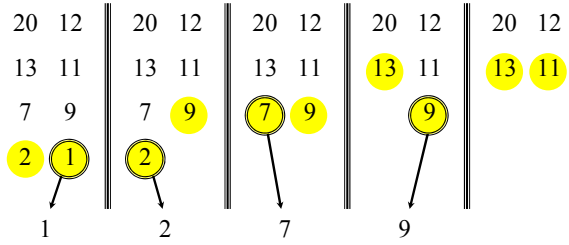
Merging two sorted arrays



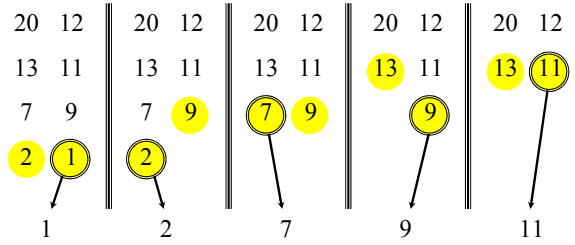
Merging two sorted arrays



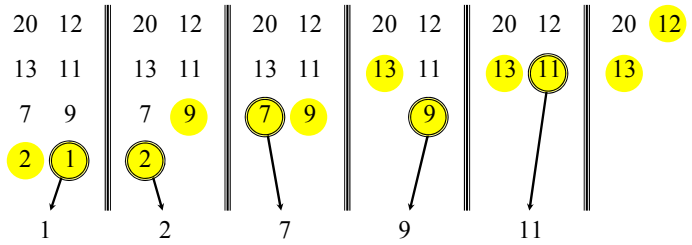
Merging two sorted arrays



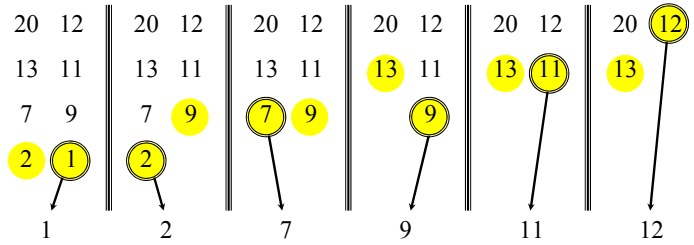
Merging two sorted arrays



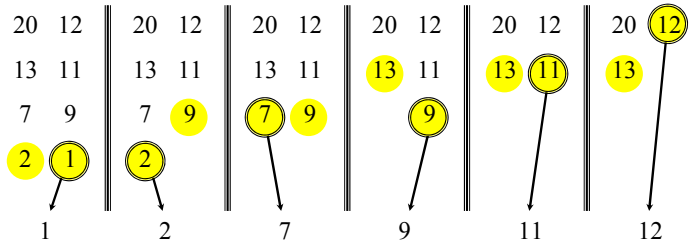
Merging two sorted arrays



Merging two sorted arrays

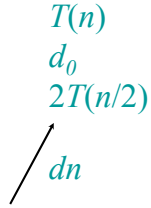


Merging two sorted arrays



Time $dn \in \Theta(n)$ to merge a total of n elements (linear time).

Analyzing merge sort



MERGE-SORT $A[1 \,..\, n]$

1. If $n = 1$, done.
2. Recursively sort $A[\,1 \,..\, \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \,..\, n\,]$.
3. ***"Merge"*** the 2 sorted lists

Sloppiness: Should be $T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor)$, but it turns out not to matter asymptotically.

Recurrence for merge sort

$$T(n) = \begin{cases} d_0 \text{ if } n = 1; \\ 2T(n/2) + dn \text{ if } n > 1. \end{cases}$$

- But what does $T(n)$ solve to? I.e., is it $O(n)$ or $O(n^2)$ or $O(n^3)$ or …?

Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.

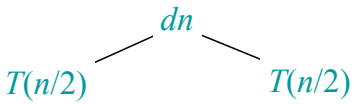
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.

$T(n)$

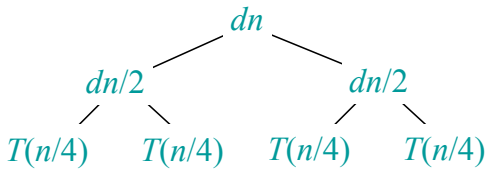
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



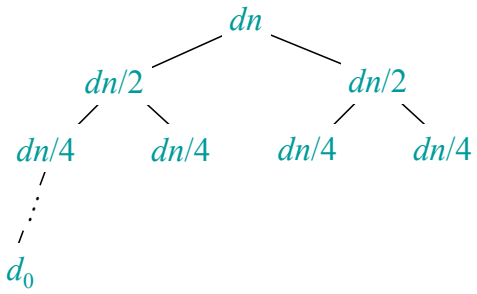
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



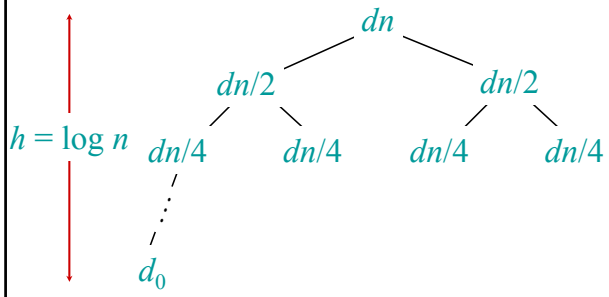
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



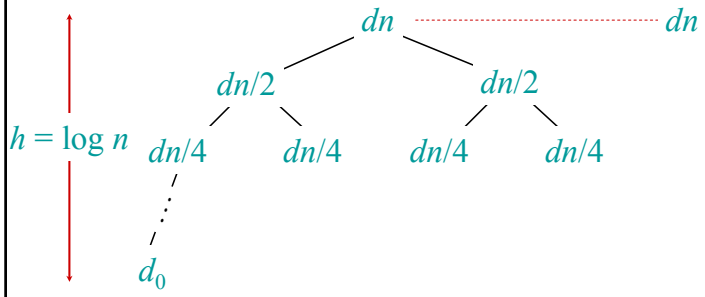
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



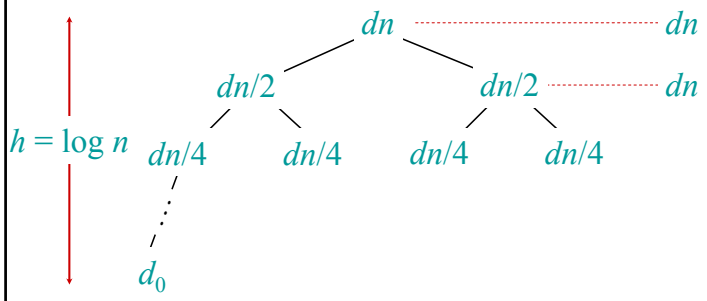
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



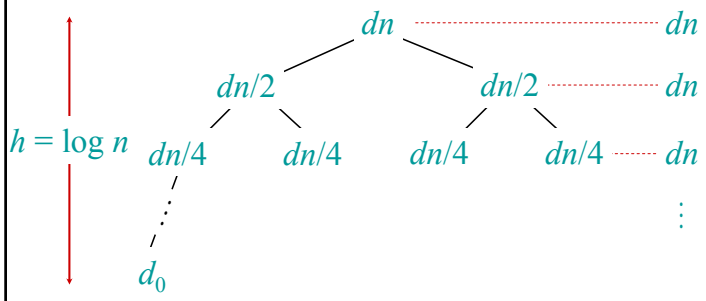
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



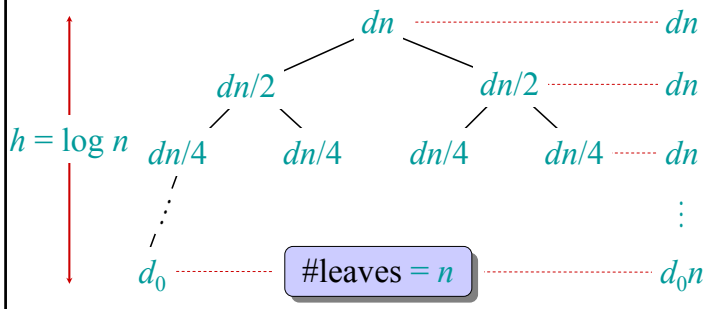
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



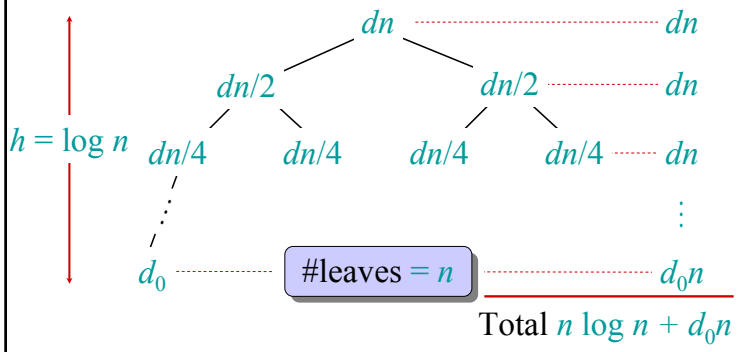
Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



Recursion tree

Solve $T(n) = 2T(n/2) + dn$, where $d > 0$ is constant.



Conclusions

- Merge sort runs in $\Theta(n \log n)$ time.
- $\Theta(n \log n)$ grows more slowly than $\Theta(n^2)$.
- Therefore, merge sort asymptotically beats insertion sort in the worst case.
- In practice, merge sort beats insertion sort for $n > 30$ or so. (Why not earlier?)

Recursion-tree method

- A recursion tree models the costs (time) of a recursive execution of an algorithm.
- The recursion-tree method can be unreliable, just like any method that uses ellipses (…).
- It is good for generating **guesses** of what the runtime could be.

But: Need to **verify** that the guess is right.
→ Induction (substitution method)

Substitution method

The most general method to solve a recurrence (prove O and Ω separately):

1. ***Guess*** the form of the solution:
   (e.g. using recursion trees, or expansion)
2. ***Verify*** by induction (inductive step).
3. ***Solve*** for O-constants n_0 and c (base case of induction)