CS 3343 Analysis of Algorithms – Spring 09

2/24/09

5. Homework

Due: Tuesday 3/3/09 before class

Justify all your answers.

1. **Deterministic Quicksort (6 points)**
   Let "Deterministic Quicksort" be the non-randomized Quicksort which takes the first element as a pivot, using the partition routine that we covered in class on slide 4 of the quicksort slides.

   (a) (4 points) In the best case the pivot always splits the array in half, for all recursive calls of Deterministic Quicksort. Give a sequence of 3 distinct numbers, a sequence of 7 distinct numbers, and a sequence of 15 distinct numbers that cause this best-case behavior. (For the sequence of 15 numbers the first two recursive calls should be on sequences of 7 numbers each, and the next recursive calls on sequences of 3 numbers).

   (b) (2 points) The runtime analyses in class were for Quicksort on an array with distinct elements. Assume now there is an array given that contains the same number n times (so for example: $2, 2, 2, \ldots, 2$). What is the runtime of Deterministic Quicksort?

2. **Decision Tree (6 points)**
   The decision tree that we had in class was for insertion sort. Draw the decision tree for mergesort of three elements. Assume that the first split occurs after the first element (so, the topmost "left" recursive call will be for only one element, and the topmost "right" recursive call will be for two elements). Remember that the comparisons only take place in the merge routine. Annotate the tree with comments as to which algorithm part a comparison belongs to.

4. **Randomized code snippet (6 points)**
   Consider the following code snippet, where `RandomInteger(i)` takes $O(1)$ time and returns an integer between 1 and i, each with probability $1/i$.

```
for(i=1; i<=n; i++){
  if(RandomInteger(i)==i){
    for(j=1; j<=n; j++){
      print(``hello'');
    }
  }
}
```

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a) (1 point) What is the best case runtime for this code

snippet? Describe what triggers a best-case scenario.

b) (1 point) What is the worst case runtime for this code snippet? Describe what triggers a worst-case scenario.

c) (4 points) Now analyze the **expected** runtime. Note that the sample space for the whole code snippet contains sequences of n numbers. Clearly define your random variable. *Hint: Break your random variable into multiple random variables.*