CS 3343 Analysis of Algorithms – Spring 09

2/10/09

4. Homework

Due **2/17/09** before class

1. **Multiplying polynomials (10 points)**
   A polynomial of degree n is a function

$$p(x) = \sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

   where a_i are constants and $a_n \neq 0$. For example, $4x^4 + 3x^2 + 2x + 1$ is a polynomial of degree 4. For simplicity you may assume that n is a power of 2.

   (a) (1 point) What is the runtime of the straight-forward algorithm for multiplying two polynomials of degree n?

   (b) (5 points) We can rewrite the polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ as $x^{n/2}(a_n x^{n/2} + a_{n-1}x^{n/2-1} + \cdots + a_{n/2+1} + a_{n/2}) + (a_{n/2-1}x^{n/2-1} + \cdots + a_1 x + a_0)$. Use this as a starting point to design a divide-and-conquer algorithm for multiplying two degree-n polynomials (recurse on polynomials of degree $n/2$). Give a runtime ana lysis of your algorithm by setting up and solving a recurrence. The runtime of your algorithm should be the same as the runtime of part a).

   (c) (1 point) Show how to multiply two degree-1 polynomials $ax + b$ and $cx + d$ using only three multiplications. *Hint: One of the multiplications is* $(a + b) \cdot (c + d)$.

   (d) (3 points) Design a divide-and-conquer algorithm for multiplying two polynomials of degree n in time $\Theta(n^{\log_2 3})$. *Hint: Reuse part* **b)** *and speed it up with the knowledge of part* **c)**

2. **Rolling dice (6 points)**
   Clearly describe the sample space and the random variables you use. 2 points will be given for correct notation.

   (a) (4 points) Suppose two fair 6-sided dice are rolled (i.e., each of the six sides is equally likely to show). Use linearity of expectation to compute the expected value of the sum of the two values showing on the rolled dice.

   (b) (2 points) Now suppose two fair k-sided dice are rolled, where $k \geq 2$ is an integer. What is the expected value of the sum of the two values showing on the rolled dice? Your answer should depend on k.

3. SimpleRoulette **(4 points)**

   The game SimpleRoulette is played as follows: The roulette wheel has a slot for each number from 1 to 36 as well as a slot for 0 and for 00. You can bet on any number between 1 and 36, but not on 0 or 00. A bet costs you \$10. If the ball drops on the slot with your number, you get paid \$360, otherwise you don't get paid anything.

Assuming that the wheel is fair (i.e., all numbers are equally likely), what is your expected win/loss in this game?

Clearly describe the sample space and the random variables you use. 2 points will be given for correct notation.