CS 3343 Analysis of Algorithms – Spring 09

4/21/09

10. Homework

Due **Tuesday 4/28/09** before class

1. **(5 points) Bellman-Ford**
   Given a weighted, directed graph $G = (V, E)$ that possibly has negative weights but that has no negative weight cycles. For given two vertices u, v, let $l(u, v)$ be the minimum number of edges in a shortest path from u to v (where the shortest path is of course based on the edge weights). Now, define k to be the maximum, over all pairs of vertices $u, v \in V$, of $l(u, v)$.

   Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $k + 1$ passes. **Do not assume that k is known in advance.**

2. **(6 points) To be or not to be ... in NP**
   Which of the problems below are in NP, and which are not? Either justify why the problem is not in NP, or show that it is in NP by sketching an appropriate algorithm and its runtime.

   a) Given three strings A, B, C. Is C a longest common subsequence of A and B?
   b) Compute a minimum spanning tree of an undirected weighted graph.
   c) Given a positive integer a, is a not a prime number (i.e., is a the product of two integers greater than 1)?

3. **(5 points) Clique and Vertex Cover**
   Show that Vertex Cover is polynomial time reducible to Clique (i.e., Vertex Cover $\leq$ Clique).

4. **(6 points) Fun with reductions**
   Suppose Π_1 and Π_2 are decision problems and Π_1 is polynomial time reducible to Π_2, so, $\Pi_1 \leq \Pi_2$. Please answer each of the questions below, and justify your answers.

   a) If $\Pi_2 \in P$ does this imply that $\Pi_1 \in P$?
   b) If $\Pi_1 \in P$ does this imply that $\Pi_2 \in P$?
   c) If Π_1 is NP-complete, does this imply that Π_2 is NP-complete?
   d) If Π_2 is NP-complete, does this imply that Π_1 is NP-complete?
   e) If $\Pi_2 \notin P$ does this imply that $\Pi_1 \notin NP$?
   f) If $\Pi_1 \in NP$ does this imply that Π_2 is NP-complete?