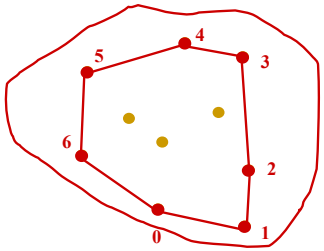
Convex Hull

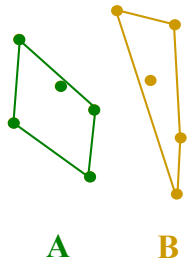
- Given a set of pins on a pinboard
- And a rubber band around them
- How does the rubber band look when it snaps tight?



- We represent the convex hull as the sequence of points on the convex hull polygon, in counter-clockwise order.

Convex Hull: Divide & Conquer

- Preprocessing: sort the points by x-coordinate
- Divide the set of points into two sets **A** and **B**:
  - **A** contains the left $\lfloor n/2 \rfloor$ points,
  - **B** contains the right $\lceil n/2 \rceil$ points
- Recursively compute the convex hull of **A**
- Recursively compute the convex hull of **B**
- Merge the two convex hulls



Convex Hull: Runtime

Preprocessing: sort the points by x-coordinate	$O(n \log n)$ just once
Divide the set of points into two sets **A** and **B**: **A** contains the left $\lfloor n/2 \rfloor$ points, **B** contains the right $\lceil n/2 \rceil$ points	$O(1)$
Recursively compute the convex hull of **A**	$T(n/2)$
Recursively compute the convex hull of **B**	$T(n/2)$
Merge the two convex hulls	$O(n)$

Convex Hull: Runtime

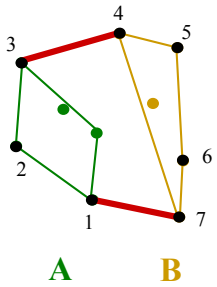
- Runtime Recurrence:

  $T(n) = 2\,T(n/2) + cn$

- Solves to $T(n) = \Theta(n \log n)$

Merging in O(n) time

- **Find upper and lower tangents** in O(n) time
- Compute the convex hull of A∪B:
  - walk counterclockwise around the convex hull of A, starting with left endpoint of lower tangent
  - when hitting the left endpoint of the upper tangent, cross over to the convex hull of B
  - walk counterclockwise around the convex hull of B
  - when hitting right endpoint of the lower tangent we're done
  - This takes O(n) time



Finding the lower tangent in O(n) time

a = rightmost point of A

b = leftmost point of B

while T=ab not lower tangent to both convex hulls of A and B do{

  while T not lower tangent to convex hull of A do{

    a=a-1

  }

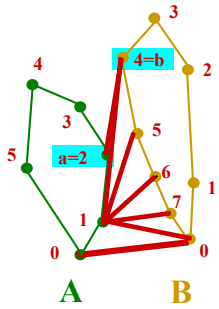
  while T not lower tangent to convex hull of B do{

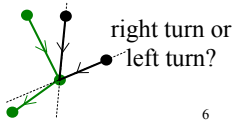
    b=b+1

  }

}

can be checked in constant time





Convex Hull: Runtime

• Preprocessing: sort the points by x-coordinate	O(n log n) just once

• Divide the set of points into two sets **A** and **B**:
  • **A** contains the left ⌊n/2⌋ points,
  • **B** contains the right ⌈n/2⌉ points
	O(1)

• Recursively compute the convex hull of **A**	T(n/2)
• Recursively compute the convex hull of **B**	T(n/2)
• Merge the two convex hulls	O(n)

Convex Hull: Runtime

- Runtime Recurrence:

  T(n) = 2 T(n/2) + cn

- Solves to T(n) = Θ(n log n)