

1. Homework

Due: Tuesday 1/31/06 before class, or **Excuse:** Thursday 2/2/06 before class

1. Code snippets

Give the Θ -runtime depending on n for the code snippet below. Justify your answer.

```
for(i=n; i>=1; i=i/2){
  for(j=1; j*j<=n; j++){
    print('Algorithms rock. ');
  }
}
```

2. Big-Oh

Use the definition of O to prove the following:

If $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$ then $f(n) \in O(h(n))$.

3. Big-Oh ranking (12 points)

Rank the following functions by order of growth, i.e., find an arrangement f_1, f_2, \dots of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3), \dots$. Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

$$n^2, n^3, \log n, 2^n, n \log n, 2^{n+1}, 3^n$$

Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the **Rule of l'Hôpital** which states that

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{f'(n)}{g'(n)}$$

if the limits exist; where $f'(n)$ and $g'(n)$ are the derivatives of f and g , respectively.