CS 3343 Analysis of Algorithms – Fall 11

10/25/11

8. Homework

Due: Tuesday 11/1/11 before class

Justify all your answers.

1. **LCS traceback (4 points)**
   Give pseudocode that performs the traceback to construct an LCS from a filled dynamic programming table *without* using the "arrows", in $O(n+m)$ time.

2. **DP in less space (4 points)**

   Suppose we only want to compute the *length* of an LCS of two strings of length m and n. Describe how to alter the dynamic programming algorithm such that it only needs $O(\min(m,n))$ space. *(Hint: Try to first develop an algorithm that runs in either $O(m)$ or $O(n)$ space, and then figure out how to cut the space down to $O(\min(m,n))$.)*

3. **Binomial coefficient (10 points)**
   Given n and k with $n \geq k \geq 0$, we want to compute the binomial coefficient $\binom{n}{k}$.

   (a) (5 points) Give pseudo-code for the bottom-up dynamic programming algorithm to compute $\binom{n}{k}$ using the recurrence

   $$\begin{aligned} \binom{n}{k} &= \binom{n-1}{k-1} + \binom{n-1}{k}, \text{ for } n > k > 0 \\ \binom{n}{0} &= \binom{n}{n} = 1, \text{ for } n \geq 0 \end{aligned}$$

   (b) (1 point) What are the runtime and the space complexity of your algorithm, in terms of n and k?

   (c) (4 points) Now assume you use memoization to compute $\binom{4}{3}$ using the above recurrence. In which order do you fill the entries in the DP-table? Give the DP-table for this case and annotate each cell with a "time stamp" (i.e., with a number $1, 2, 3, ...$) when it was filled.

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Practice Problems

(Not required for homework credit.)

1. **DP in less space**

   The bottom-up dynamic programming algorithm computing the n-th Fibonacci number $F(n)$ takes $O(n)$ time and uses $O(n)$ space. Show how to modify the algorithm to use only constant space.

2. **Binomial coefficient**

   (a) What is another definition of the binomial coefficient $\binom{n}{k}$?

   (b) Show how $\binom{4}{3}$ is computed using bottom-up dynamic programming based on the following recurrence:

$$\begin{aligned}\binom{n}{k} &= \binom{n-1}{k-1} + \binom{n-1}{k}, \text{ for } n > k > 0 \\ \binom{n}{0} &= \binom{n}{n} = 1, \text{ for } n \geq 0\end{aligned}$$

3. **Memoization**

   Compute the length of LCS("ABC'', "BAC'') using memoization. In which order do you fill the entries in the DP-table? Give the DP-table for this case and annotate each cell with a "time stamp" (i.e., with a number $1, 2, 3, ...$) when it was filled.