CS 3343 Analysis of Algorithms – Fall 11

9/20/11

4. Homework

Due **9/27/11** before class

1. **Master theorem (10 points)**
   Use the master theorem to solve the following recurrences. Justify your results. Assume that $T(1) = 1$.

   (a) $T(n) = 2T(\frac{n}{2}) + n^2$
   (b) $T(n) = 64T(\frac{n}{4}) + \sqrt{n}$
   (c) $T(n) = 9T(\frac{n}{3}) + n^2 \log n$
   (d) $T(n) = 32T(\frac{n}{2}) + n^2 \log n$

2. **Strassen's Algorithm (10 points)**
   Apply Strassen's algorithm to compute

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

   The recursion should exit with the base case $n = 1$, i.e., 2×2 matrices should recursively be computed using Strassen's algorithm.

   You can use the results from the practice problem on the back of this page, i.e., you can assume that certain products and sub-matrices have been computed already as part of the practice problem.

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Practice Problems

(Not required for homework credit.)

1. **Master theorem**
   Use the master theorem to solve the following recurrences. Justify your results. Assume that $T(1) = 1$.

   (a) $T(n) = 125T(\frac{n}{5}) + 1$
   (b) $T(n) = 16T(\frac{n}{4}) + n^4$
   (c) $T(n) = 2T(\frac{n}{4}) + \sqrt{n}\log n$

2. **Matrix Multiplication**
   Consider computing the following matrix product:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

   (a) Compute the product using the regular matrix multiplication algorithm. In which order did you compute the entries in the result matrix?
   (b) Now consider applying a straight-forward divide-and-conquer algorithm to compute the product (not Strassen's algorithm yet). The recursion should exit with the base case $n = 1$ (which just multiplies two numbers). Which sub-matrices do you need to multiply for this? Outline your steps.
   (c) Now consider applying Strassen's algorithm to compute the product. The recursion should exit with the base case $n = 1$ (i.e., 2×2 matrices should recursively be computed using Strassen's algorithm).

   For this practice question, only compute the upper right quadrant as well as the lower left quadrant of the result matrix. (The whole product should be computed as part of the homework.)