CS 3343 Analysis of Algorithms – Fall 11

8/30/11

1. Homework

Due **9/6/11** before class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Bubblesort (10 points)**

   Bubblesort is a sorting algorithm that works by repeatedly swapping adjacent elements that are out of order. Consider the version of Bubblesort below which sorts the array $A[1..n]$ into increasing order by repeatedly "bubbling the minimum element of $A[i+1..n]$ to the left".

```
Bubblesort(A[1..n]){
  for(i=1; i<=n; i++)
    //Bubble min of A[i+1..n] to the left
    for(j=n; j>=i+1; j--)
      if(A[j]<A[j-1])
        swap A[j} with A[j-1]
}
```

   Let $A'[1..n]$ denote the output of `Bubblesort(A)`. To prove that Bubblesort is correct, we need to prove that $A'[1] \leq A'[2] \leq \ldots \leq A'[n]$.

   (a) (2 points) State a loop invariant for the outer for-loop that will allow you to prove the correctness of the algorithm.

   (b) (4 points) Use the loop invariant to prove the correctness of the algorithm. For this you need to prove by induction that the loop invariant holds for all iterations of your loop ("base step" and "inductive step"), and then use the loop invariant in the "termination step" to prove the correctness of the algorithm.

   (c) (4 points) Give best-case and worst-case running times in asymptotic (i.e., big-Oh) notation. Also give example inputs attaining these runtimes.

2. O, Ω, Θ **(16 points)**

   Show using the definitions of big-Oh, Ω, and Θ:

   (a) (2 points) $3n^4 - 5n^3 + 4n^2 - 2 \in O(n^4)$

   (b) (4 points) $4n^3 + 3n + 2 \in \Theta(n^3)$

   (c) (2 point) Is $2^{n+1} \in O(2^n)$? Justify your answer.

   (d) (2 point) Is $2^{2n} \in O(2^n)$? Justify your answer.

   (e) (6 points) Which of the following statements is true? Justify your answers. If $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$ then

      i. $f_1(n) + f_2(n) \in O(\min(g_1(n), g_2(n)))$.

      ii. $f_1(n) + f_2(n) \in O(\max(g_1(n), g_2(n)))$.

      iii. $f_1(n) + f_2(n) \in O(g_1(n) + g_2(n))$.