CS 3343 Analysis of Algorithms – Fall 10

10/19/10

7. Homework

Due: Tuesday 10/26/10 before class

Justify all your answers.

1. **Red-black tree rotations (6 points)**
   Find a sequence of numbers which, when incrementally inserted into a red-black tree, causes the following sequence of rotations:

   left, right, right, right.

   You may start with an initially non-empty tree, and you may insert numbers that do not cause any rotations. But there should not be any additional rotations performed.

   Draw the sequence of trees that you obtain after each insertion. For each such tree indicate the node that violates the red-black tree condition, indicate the nodes that participate in the rotation, the type of the rotation, and the subtrees that correspond to each other before and after the rotation.
   Hint: Use a red-black tree demo from the web.

2. **Black-height (6 points)**
   The *black-height* of a red-black tree is the black-height of its root vertex.

   (a) What is the largest possible number of keys that a red-black tree of black-height b can store? Your answer should depend on b.
   (b) What is the smallest possible number of keys that a red-black tree of black-height b can store? Your answer should depend on b.

   Justify your answers.

3. **Keys in a B-tree (3 points)**
   What is the maximum number of keys that can be stored in a B-tree of minimum degree k and height h? Your answer should depend on k and h.

4. **B-tree-search using binary search (4 points)**

   Consider changing B-Tree-Search to use **binary search** instead of linear search on the key.

   (a) What is the number of disk accesses?
   (b) What is the CPU time? Show that the CPU time is only $O(\log n)$, which is independent of k.

5. **Legal B-trees (4 points)**
   Describe all legal B-trees of minimum degree 2 that store all the keys $1, 2, 3, 4, 5$.