CS 3343 Analysis of Algorithms – Fall 10

9/30/10

5. Homework

Due: Tuesday 10/12/10 before class

Justify all your answers.

1. **Expected values of dice (8 points)**
   Clearly describe the sample space and the random variables you use. Half of the points will be given for proper notation.

   (a) (2 points) Compute the expected value of rolling a fair four-sided die.

   (b) Compute the expected value of the sum of two fair four-sided dies...

      i. (2 points) ... using the definition of the expected value.

      ii. (2 points) ... using linearity of expectation. *(Hint: Express your random variable as the sum of two random variables.)*

   (c) (2 points) Use linearity of expectation to compute the expected value of the sum of k fair four-sided dice, for any $k \geq 1$. *(Hint: Express your random variable as the sum of k random variables.)*

2. **Minimum in an array (8 points)**
   An array $A[1..n]$ contains n distinct numbers that are randomly ordered, with each permutation of the n numbers being equally likely. The task is to compute the expected value of the index of the minimum element in the array.

   (a) (1 point) Describe the sample space.

   (b) (1 point) Describe the random variable of interest *(Hint: We want to compute the expected value of the random variable; look at the problem statement.)*

   (c) (2 points) Consider an example array of $n = 5$ numbers. Consider four different orderings of the numbers in this array, and for each of these orderings provide the value of the random variable.

   (d) (2 points) Now consider an arbitrary n again. What is the probability that the minimum of the array is contained in the first slot? And what is the probability that it is contained in the second slot?

   (e) (2 points) Use the following definition of an expected value to compute the expected value of your random variable.

   $$E(X) = \sum_{x \in \mathbb{R}} p(X = x) * x$$

   Note that $p(X = x)$ is short for $p(\{s \in S \mid X(s) = x\})$, or in other words this is the probability that the random variable X takes one specific value x.

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3. **Best case for quicksort (5 points)**
   Let "Deterministic Quicksort" be the non-randomized Quicksort which takes the first element as a pivot, using the partition routine that we covered in class on slide 4 of the quicksort slides.

   In the best case the pivot always splits the array in half, for all recursive calls of Deterministic Quicksort. Give a sequence of 3 distinct numbers, a sequence of 7 distinct numbers, and a sequence of 15 distinct numbers that cause this best-case behavior. (For the sequence of 15 numbers the first two recursive calls should be on sequences of 7 numbers each, and the next recursive calls on sequences of 3 numbers).