CS 3343 Analysis of Algorithms – Fall 10

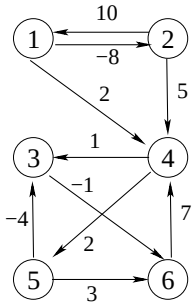
11/23/10

11. Homework

Due **Thursday 12/2/10** before class

1. **Floyd-Warshall (7 points)**

   Run the Floyd-Warshall algorithm on the weigthed directed graph below. Show the matrix $c^{(k)}$ that results for each iteration of the outer loop (for k=0,...,6).



2. **To be or not to be ... in NP (8 points)**
   Which of the problems below are in NP, and which are not? Either justify why the problem is not in NP, or show that it is in NP by sketching an appropriate algorithm and its runtime.

   (a) Given an array A of numbers. Does A represent a min-heap?

   (b) Given a positive integer a, is a not a prime number (i.e., is a the product of two integers greater than 1)?

   (c) Compute a shortest path between two vertices u, v in a directed graph with positive edge weights.

   (d) Given an array $A[1..n]$ of numbers, and a number K. Is there a subset of numbers in A that sum up to exactly K?

3. Weighted Clique **(6 points)**
   Consider the following problem:

   > Weighted Clique
   > **Given:** An undirected graph G with positive integer edge weights, and an integer $S > 0$.
   > **Task:** Does G contain a clique C with total weight (= sum of edge weights) at least S?

   Show that Weighted Clique is NP-hard by showing:

   (a) that Weighted Clique is in NP, and

   (b) by giving a polynomial-time reduction from Clique (so, show Clique $\leq$ Weighted Clique)

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4. **Fun with reductions (6 points)**
Suppose Π_1 and Π_2 are decision problems and Π_1 is polynomial time reducible to Π_2, so, $\Pi_1 \leq \Pi_2$. Please answer each of the questions below, and justify your answers.

   (a) If $\Pi_2 \in P$ does this imply that $\Pi_1 \in P$?

   (b) If $\Pi_1 \in P$ does this imply that $\Pi_2 \in P$?

   (c) If Π_2 is NP-complete, does this imply that Π_1 is NP-complete?