CS 3343 Analysis of Algorithms – Fall 07

11/20/07

10. Homework

Due **Thursday 11/29/07** before class

1. **To be or not to be ... in NP**
   Which of the problems below are in NP, and which are not? Either justify why the problem is not in NP, or show that it is in NP by sketching an appropriate algorithm and its runtime.

   a) Given a rooted binary tree T with numbers stored in the nodes. Is T a binary search tree?

   b) Compute a DFS tree of an undirected graph.

   c) Given a positive integer a, is a not a prime number (i.e., is a the product of two integers greater than 1)?

2. CLIQUE **and** VERTEX COVER
   Show that CLIQUE is NP-hard by showing:

   - that CLIQUE is in NP, and
   - by giving a reduction from VERTEX COVER (so, show VERTEX COVER $\leq$ CLIQUE)

   Please give an explicit polynomial time reduction from VERTEX COVER to CLIQUE and do not argue that we have already given a reduction from SAT to CLIQUE.

3. **Fun with reductions**
   Suppose Π_1 and Π_2 are decision problems and Π_1 is polynomial time reducible to Π_2, so, $\Pi_1 \leq \Pi_2$. Please answer each of the questions below, and justify your answers.

   a) If $\Pi_1 \in P$ does this imply that $\Pi_2 \in P$?

   b) If $\Pi_2 \in P$ does this imply that $\Pi_1 \in P$?

   c) If Π_1 is NP-complete, does this imply that Π_2 is NP-complete?

   d) If Π_2 is NP-complete, does this imply that Π_1 is NP-complete?

   e) If $\Pi_1 \in NP$ does this imply that Π_2 is NP-complete?

   f) If $\Pi_2 \notin P$ does this imply that $\Pi_2 \notin NP$?