

## 8. Homework

Due **Wednesday 11/17/04** before class

Make sure to justify your answers. Ideally, specify the sample space and the event you are calculating the probability of. (This way we may give you partial credit for partially correct answers.)

### 1. Probability rules

Remember, a probability  $P$  on a sample space  $\Omega$  is defined as a function  $P : \mathcal{P}(\Omega) \rightarrow [0, 1]$  satisfying the following two conditions:

- (i)  $P(\Omega) = 1$
- (ii)  $P(E \cup F) = P(E) + P(F)$  for all disjoint events  $E, F \in \mathcal{P}(\Omega)$

- (2 points) Using only the definition of probability, show that  $P(\emptyset) = 0$ .
- (3 points) Using only the definition of probability, show that, for any  $n \geq 2$ ,

$$P\left(\bigcup_{i=1}^n E_i\right) = \sum_{i=1}^n P(E_i)$$

for all pairwise disjoint events  $E_1, \dots, E_n \in \mathcal{P}(\Omega)$ . *Hint: Use induction on  $n$ .*

### 2. Roulette (3 points)

In roulette, a wheel with 38 numbers 0, 00, 1, 2,  $\dots$ , 36 is spun. Every number is equally likely. You bet \$10 on any number between 1 and 36. If the number comes up you win \$360. What is your expected win/loss? Make sure to specify the random variable you're using.

#### 3. 5.1 (page 360)

- (1 point) 4
- (1 point) 8
- (1 point) 22
- (1 point) 28
- (1 point) 36

#### 4. 5.2 (page 376)

- (1 point) 2
- (1 point) 6a
- (2 points) 8a *Hint: First select the positions of 1 and 2. Then select the rest of the permutation.*
- (2 points) 12

#### 5. 5.3 (page 392)

- (2 points) 2
- (3 points) 18
- (2 points) 20