CS 2233 -- Fall 2008
Discrete Mathematical Structures

1.1 Propositional Logic

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Propositions

Definition. A *proposition* is a sentence that is either true (T) or false (F), but not both.

Examples: Which of the following are propositions?
- The Alamo is located in San Antonio.
- 4+2 = 42
- UTSA is the best school in the world.
- 2*3 = 6
- 2*x = 6
- It is warm in San Antonio

Negation ¬

Definition. Let p be a proposition. The *negation* ("not") of p, denoted by $\neg p$, has the opposite truth value than the truth value of p. Read $\neg p$ as: "not p" or "It is not the case that p".

Truth Table:

p	$\neg p$
T	F
F	T

Examples: Negate the following:
- "The Alamo is located in San Antonio."

» "The Alamo is not located in San Antonio" or "It is not the case that the Alamo is located in San Antonio"
- Today is Monday

» "Today is not Monday" or "It is not the case that today is Monday"

Conjunction ∧

Definition. Let p and q be propositions. The *conjunction* ("and") of p and q, denoted by $p \wedge q$, is true when both p and q are true and is false otherwise.
Read $p \wedge q$ as: "p and q".

Truth Table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Examples: Find the conjunction of p and q:
- p: "It is sunny today." q: "Today is Monday."

» "It is sunny today and today is Monday."
The conjunction is true on sunny Mondays (TT) but it is false on any non-sunny day (FT or FF) and it is false on any other day but Monday (TF or FF).

Disjunction $\vee$

Definition. Let p and q be propositions. The *disjunction* ("inclusive or") of p and q, denoted by $p \vee q$, is false when both p and q are false and is true otherwise.
Read $p \vee q$ as: "p or q".

Truth Table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Examples: Find the disjunction of p and q:

• p: "It is sunny today." q: "Today is Monday."

» "It is sunny today or today is Monday."

The disjunction is true on sunny Mondays (*TT*) and on Mondays (*FT* or *TT*) and on sunny days (*TF* or *TT*). It is only false on non-sunny days that are not Mondays (*FF*).

Exclusive Or $\oplus$

Definition. Let p and q be propositions. The *exclusive or* ("xor") of p and q, denoted by $p \oplus q$, is true when exactly one of p and q is true, and false otherwise.
Read $p \oplus q$ as: "p xor q".

Truth Table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Where is the difference between or and xor?

• "Students who have taken calculus or biology can take this class." Is this $p \vee q$ or $p \oplus q$?

• The use of "or" in English is usually inclusive (i.e., $\vee$).

• How can we make this statement exclusive (i.e., $\oplus$)?

» "Students who have taken calculus or biology, but not both, can enroll in this class."

Note that "either….or" is supposed to be exclusive, but we often don't use it in the correct way in English.

Conditional Statement $\rightarrow$

Definition. Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is false when p is true and q is false, and true otherwise. p is called the *hypothesis* and q the *conclusion*.

Read $p \rightarrow q$ as: "if p, then q" "p implies q"
"p only if q"
….. many more examples in the book.

Truth Table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Examples:

• "If I am elected, then I will lower taxes."

» $p \rightarrow q$ with p "elected" and q "taxes"

• p: "It rains." q: "We get wet."

» "If it rains, then we will get wet."

» "We will get wet whenever it rains."

» "It rains only if we get wet."

Biconditional Statement $\leftrightarrow$

Definition. Let p and q be propositions. The *biconditional statement* ("iff") $p \rightarrow q$ is true when p and q have the same truth value, and false otherwise.

Read $p \leftrightarrow q$ as: "p if and only if q"
"p *iff* q"

Truth Table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example:

• "You can take the flight if and only if you buy a ticket."

Truth Tables for Compound Propositions

For compound propositions such as

$(\neg p \land q) \lor (q \rightarrow p)$

we can incrementally construct the truth table.

Precedence of logical operators:

highest	$\neg$
	$\land$
	$\lor$
	$\rightarrow$
lowest	$\leftrightarrow$

Truth Table:

p	q	$\neg p$	$\neg p \land q$	$q \rightarrow p$	$(\neg p \land q) \lor (q \rightarrow p)$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	F	T	T

Translating English Sentences

Examples:

• You can access the internet from campus only if you are a computer science major or you are not a freshman"

(access the internet from campus = a; only if = $\rightarrow$; you are a computer science major = c; or = $\lor$; not = $\neg$; a freshman = f)

» $a \rightarrow (c \lor \neg f)$

• You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

(not = $\neg$; ride the roller coaster = q; if = $\rightarrow$; you are under 4 feet tall = r; unless = $\land \neg$; you are older than 16 years old = s)

» $(r \land \neg s) \rightarrow \neg q$