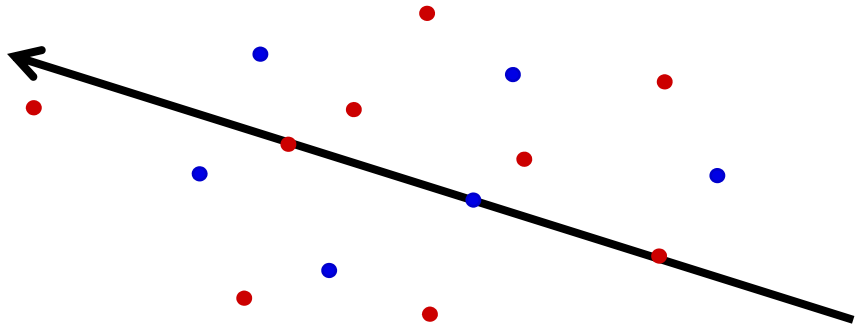
CMPS 6640/4040 Computational Geometry
Spring 2016

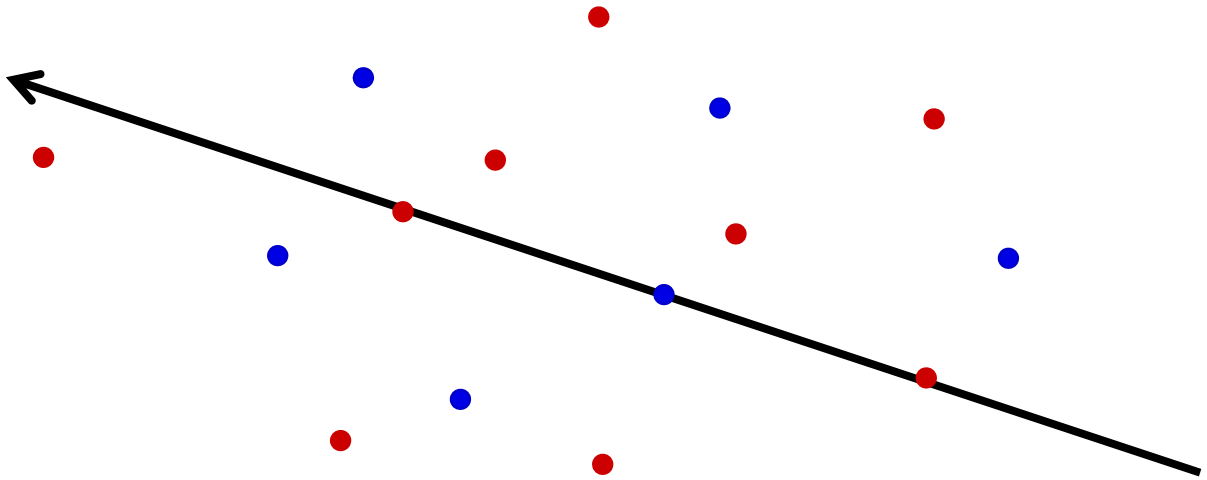


Ham Sandwich Theorem

Carola Wenk

Ham-Sandwich Theorem

Theorem: Let P and Q be two finite point sets in the plane Then there exists a line l such that on each side of l there are at most $|P|/2$ points of P and at most $|Q|/2$ points of Q.



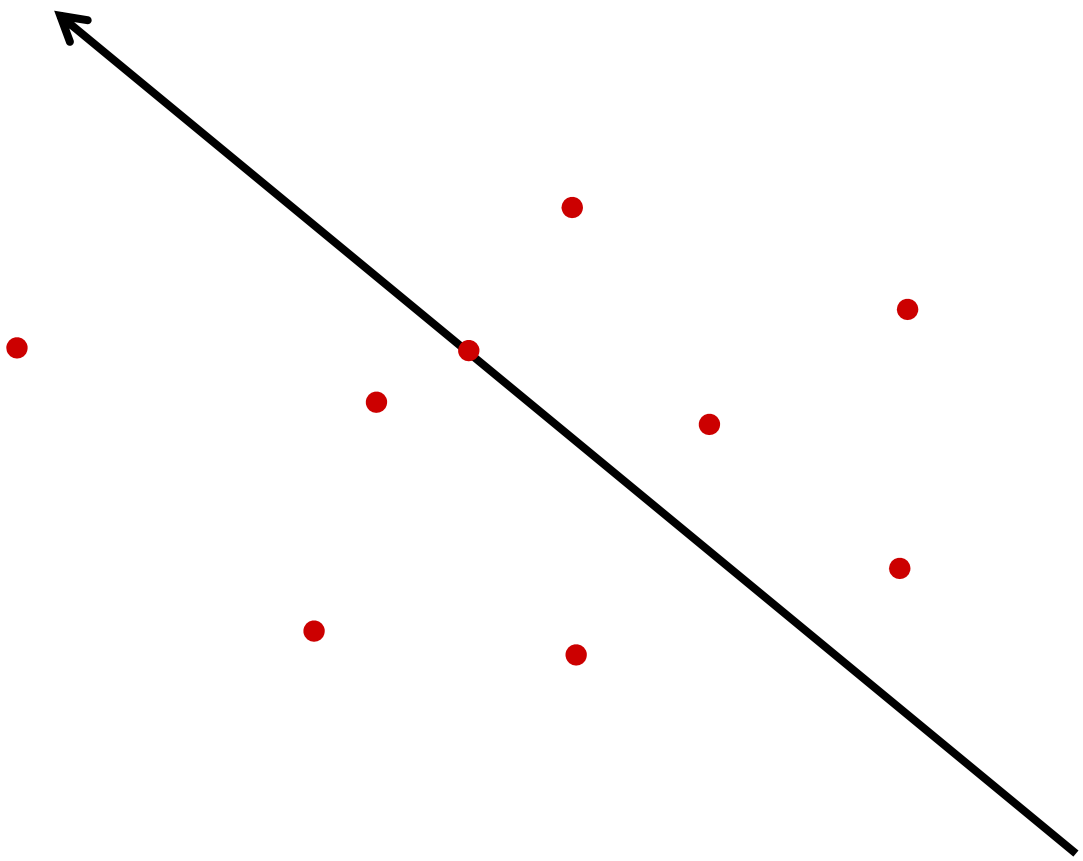
Ham-Sandwich Theorem

Proof:

Find a line l such that on each side of l there are at most $|P|/2$ points of P.

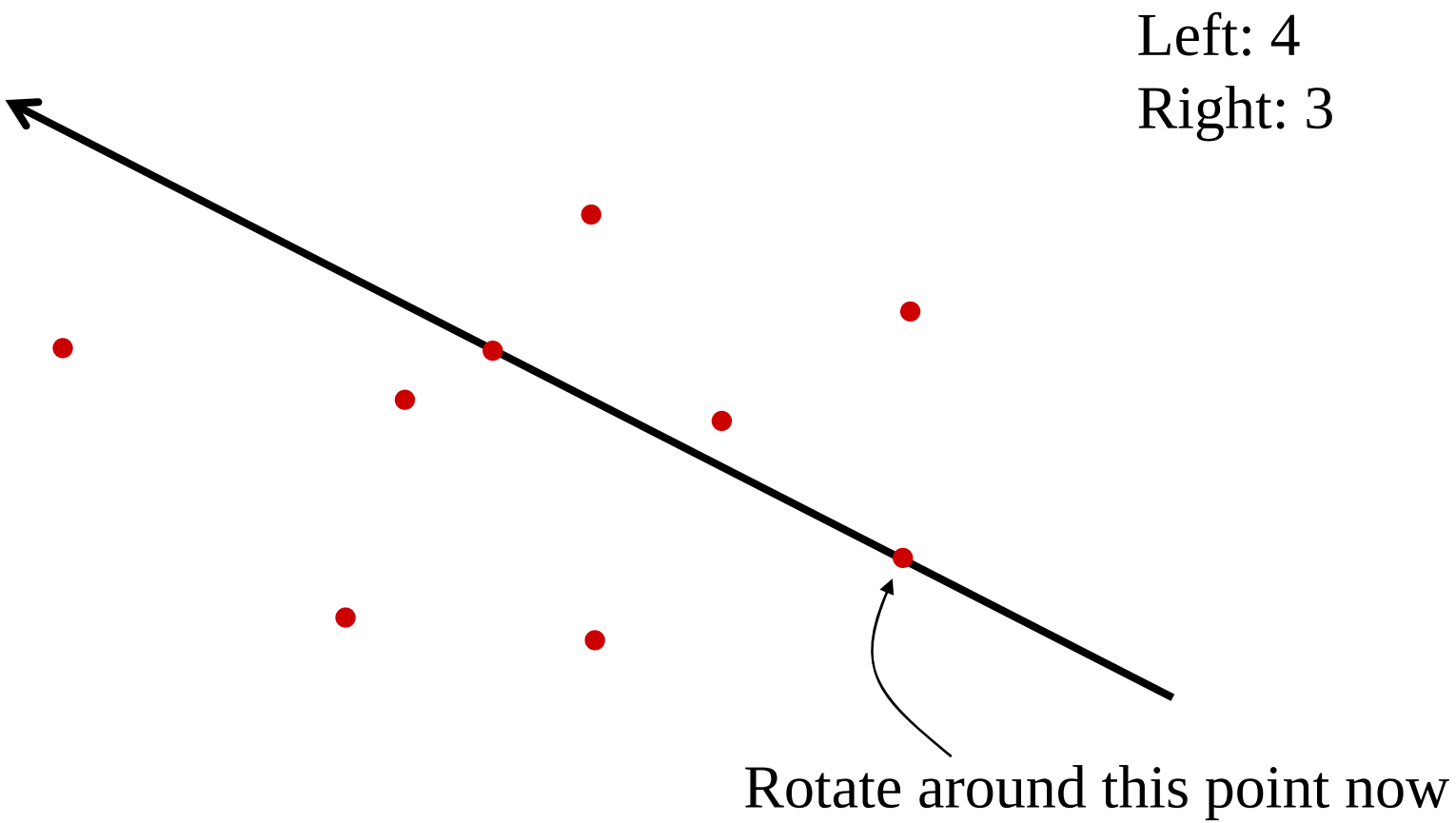
Then rotate l counter-clockwise in such a way that there are always at most $|P|/2$ points of P on each side of l.

Rotation

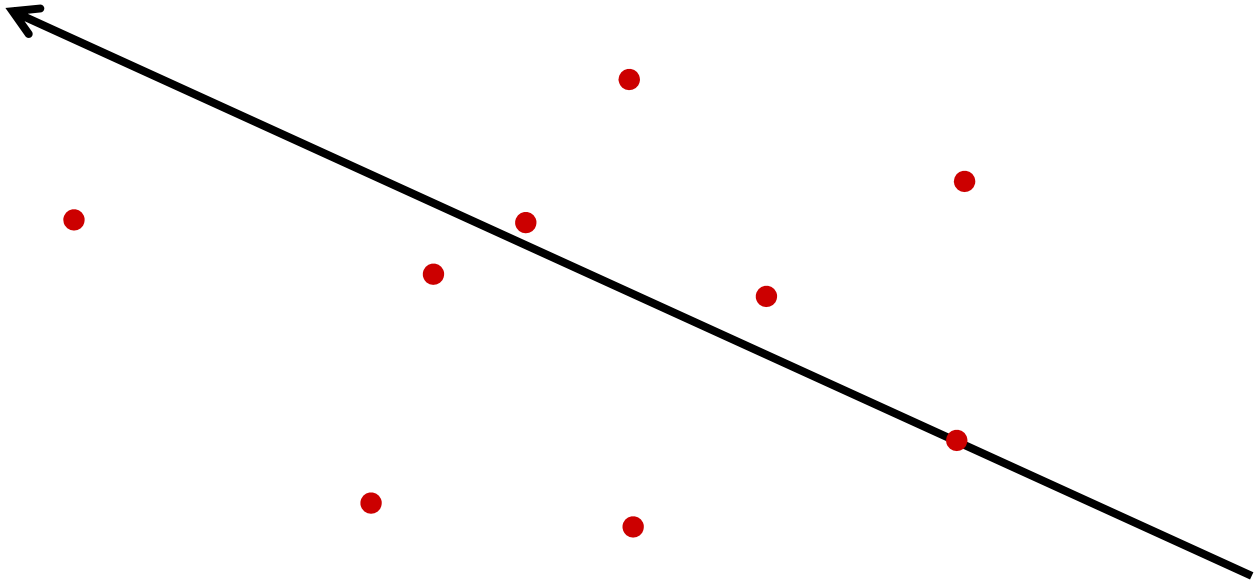


Left: 4
Right: 4

Rotation

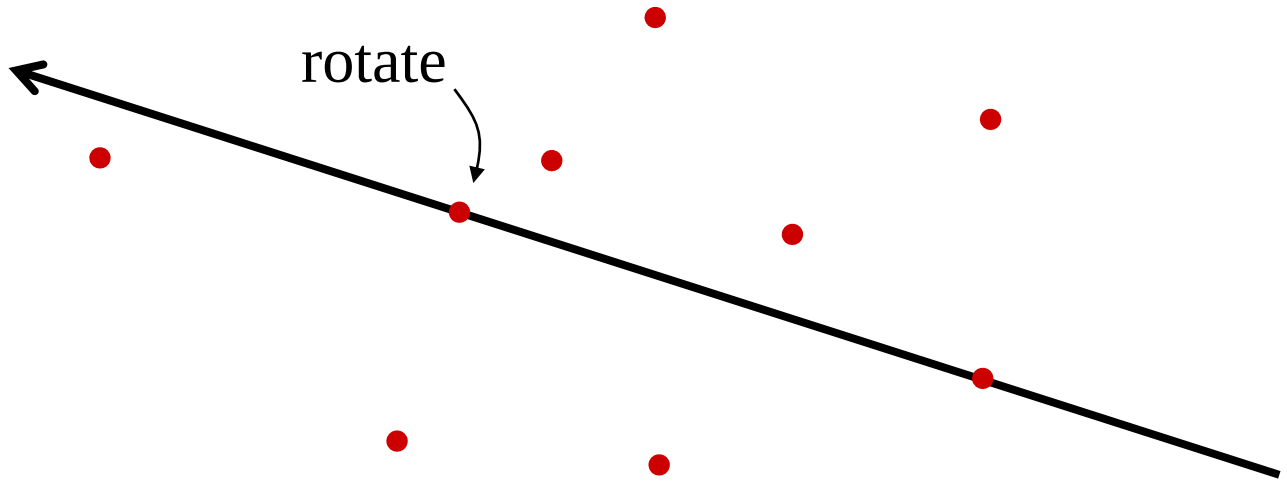


Rotation



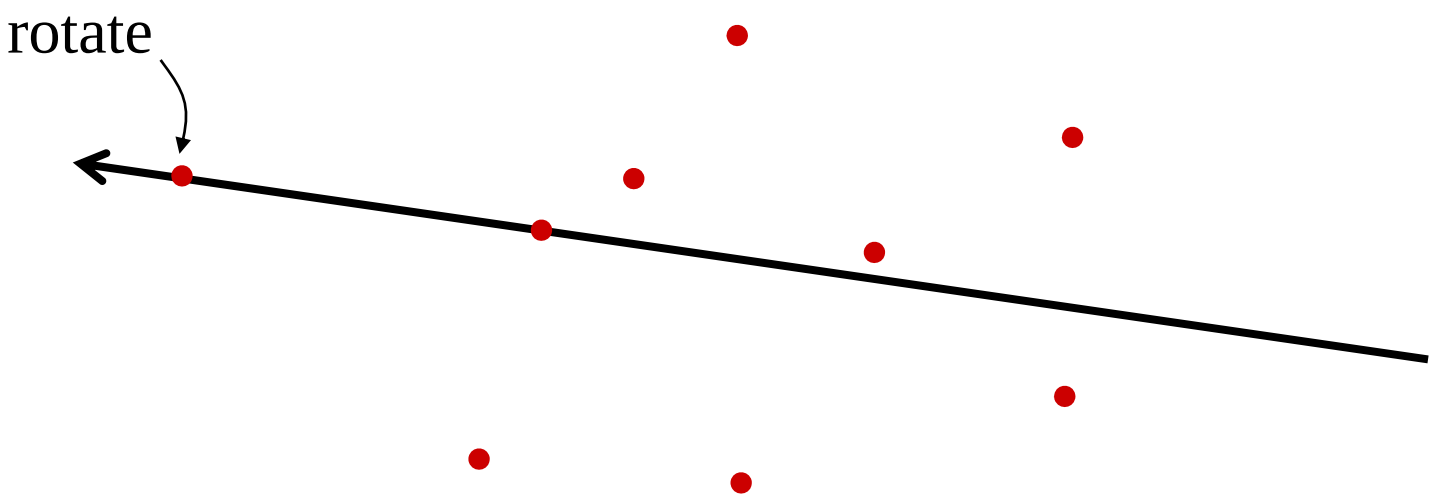
Rotation

Left: 3
Right: 4



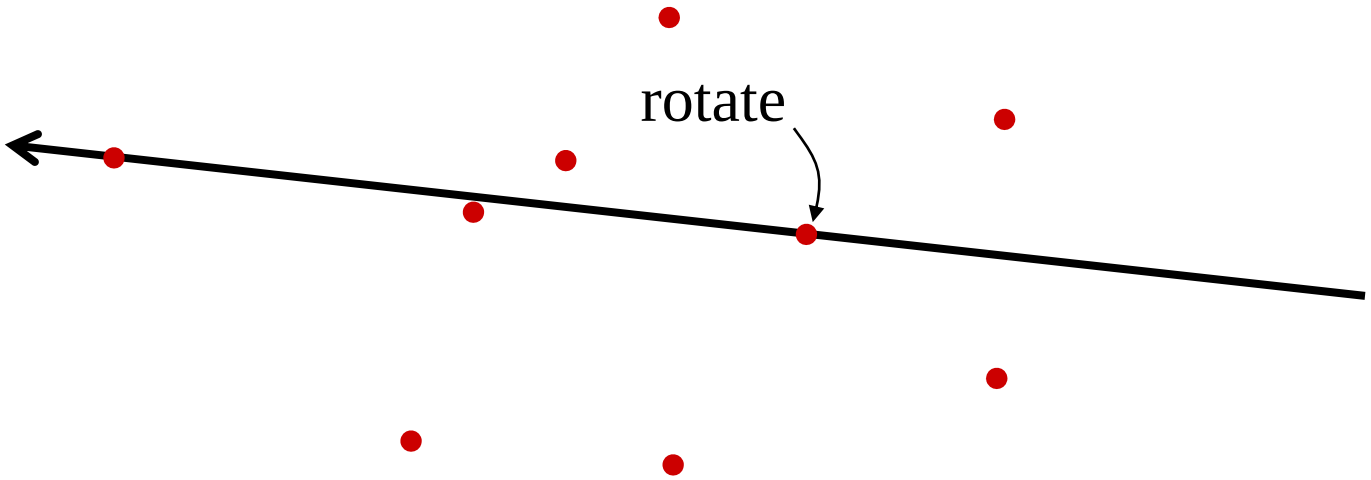
Rotation

Left: 3
Right: 4



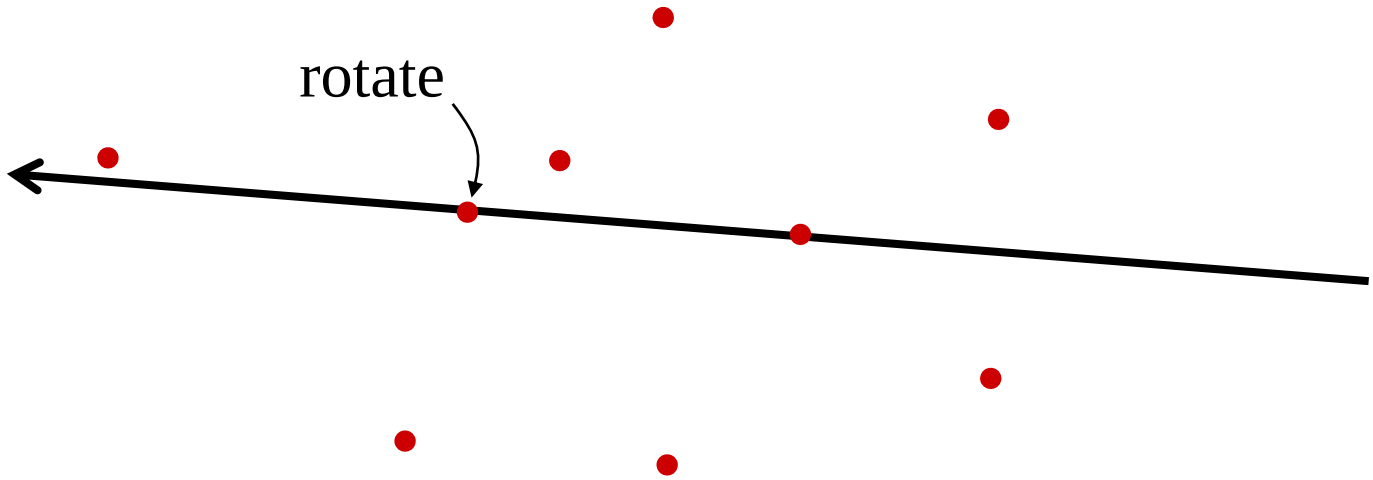
Rotation

Left: 4
Right: 3

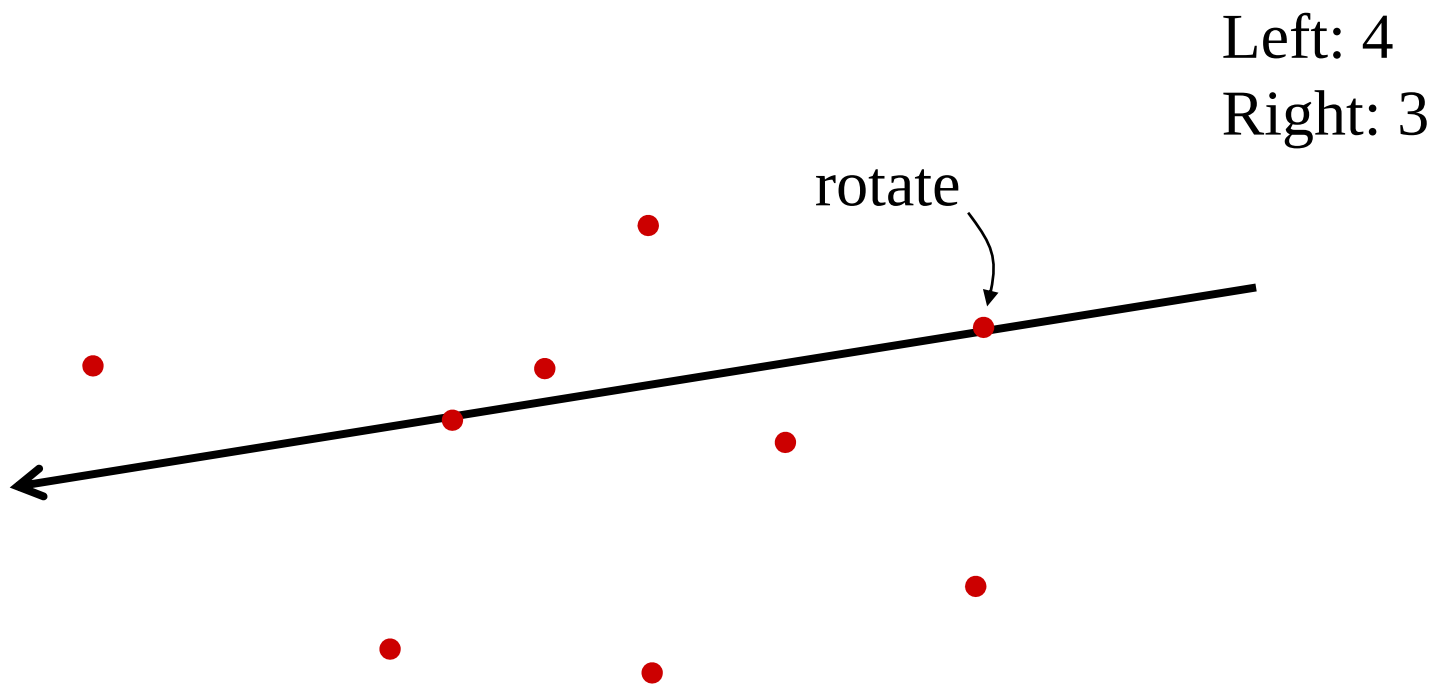


Rotation

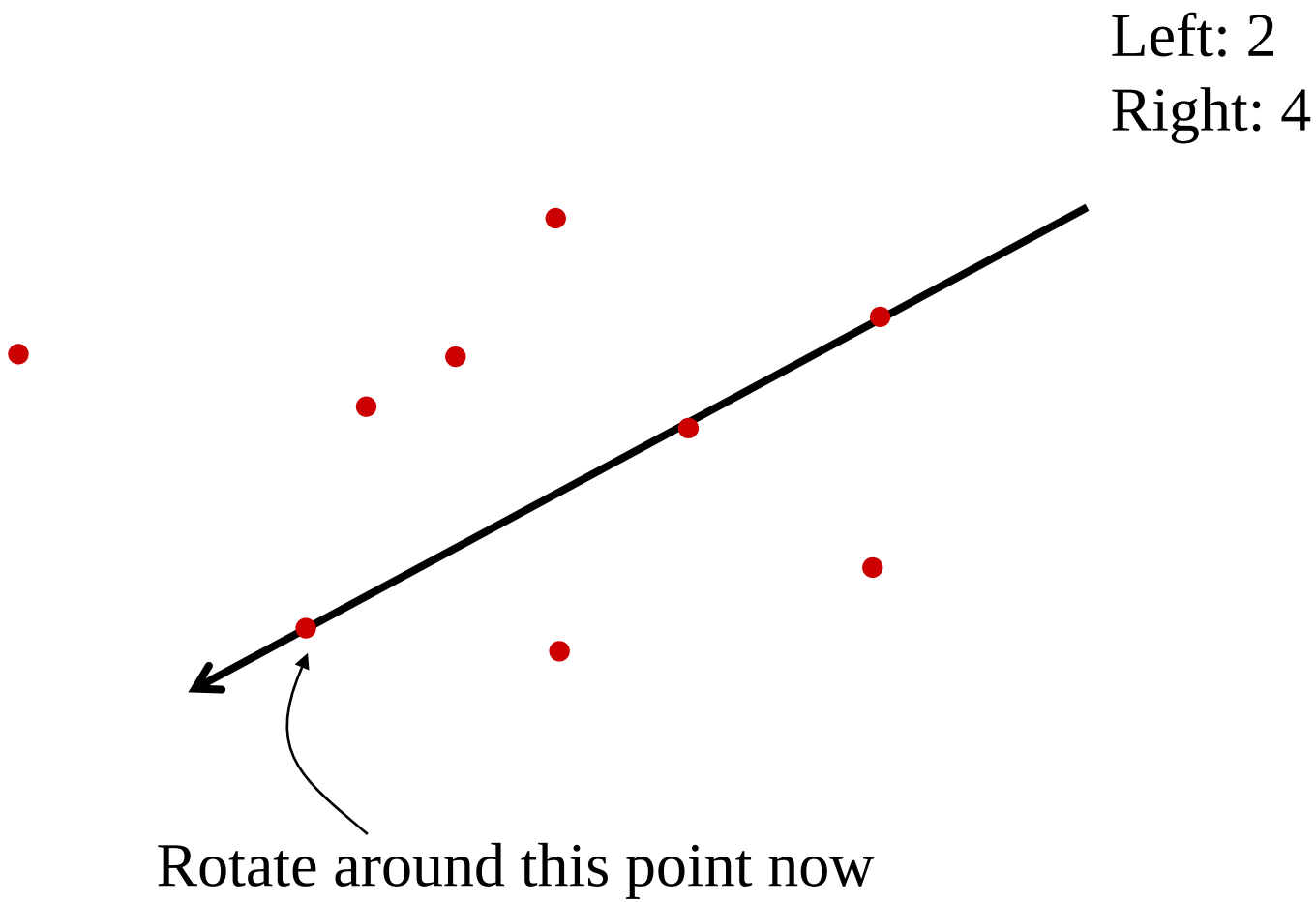
Left: 3
Right: 4



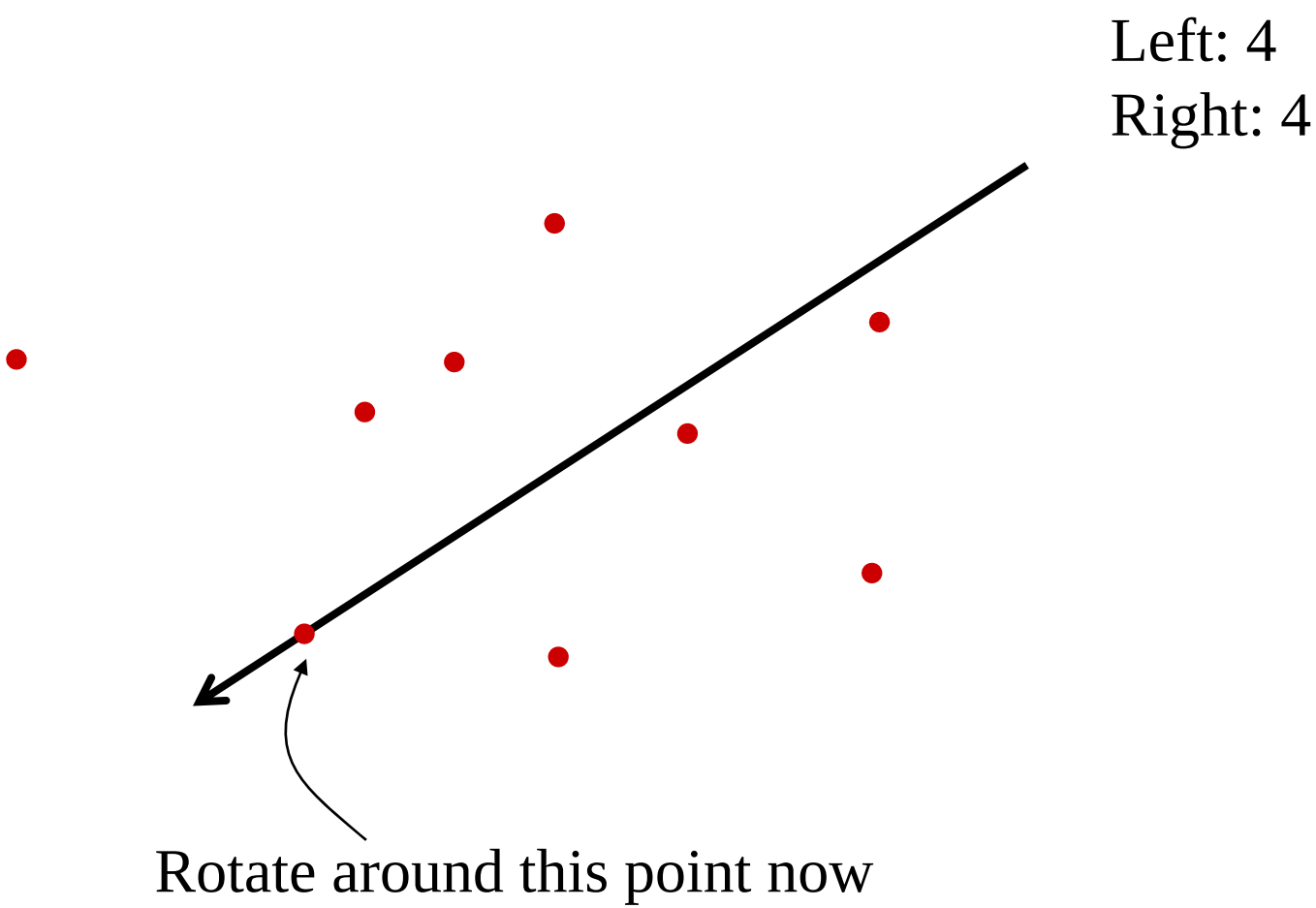
Rotation



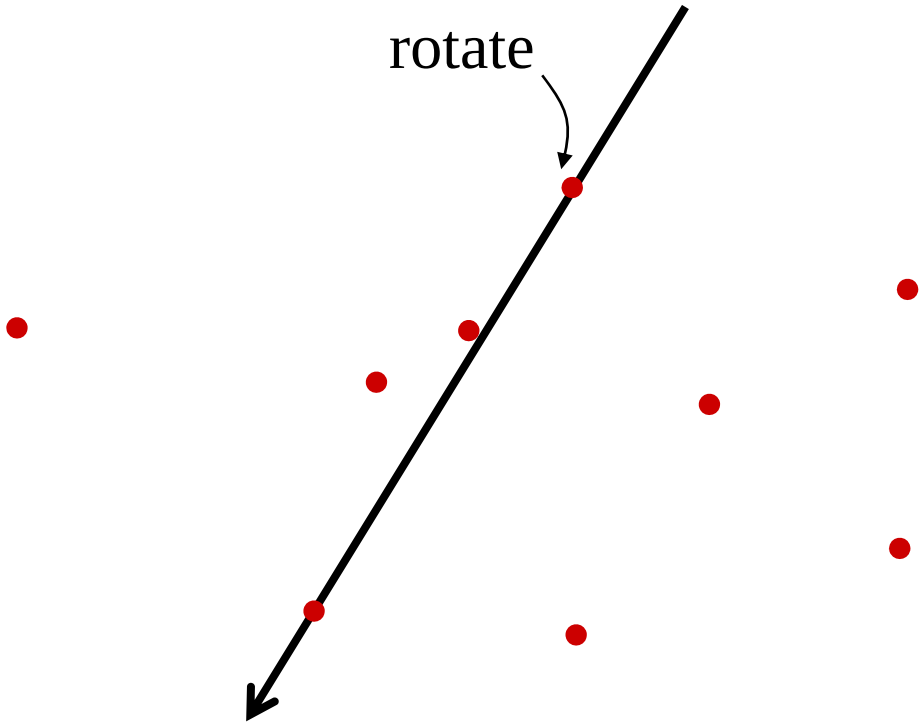
Rotation



Rotation

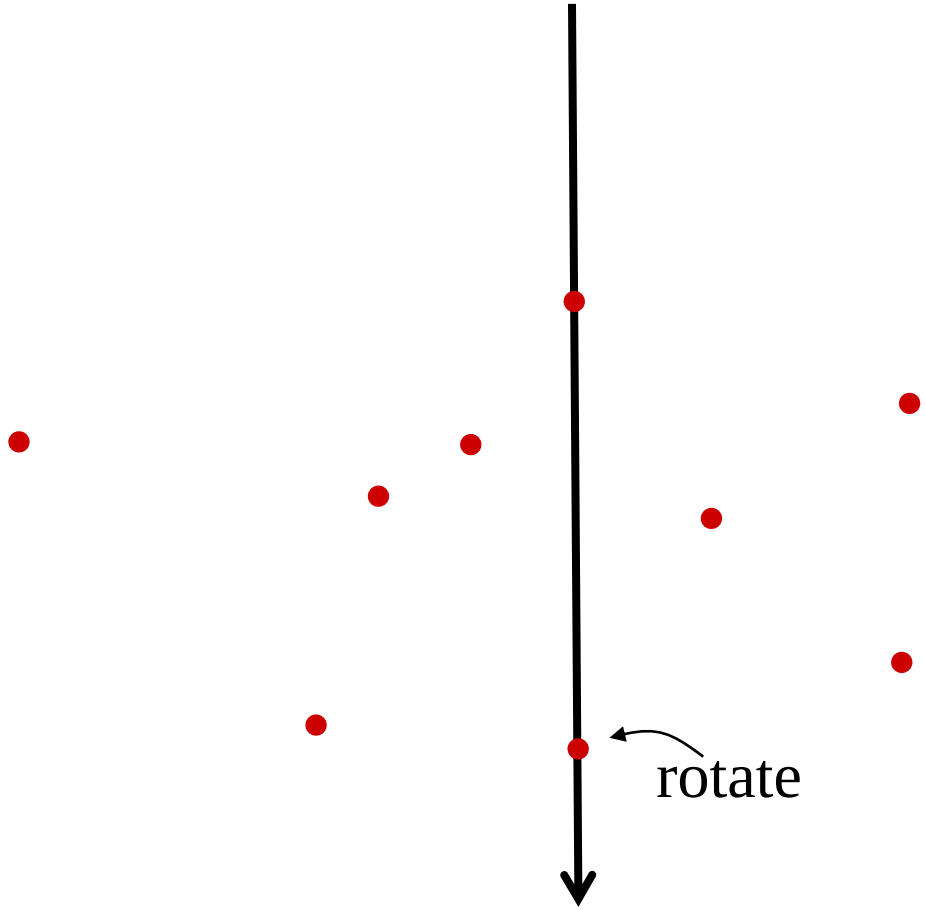


Rotation



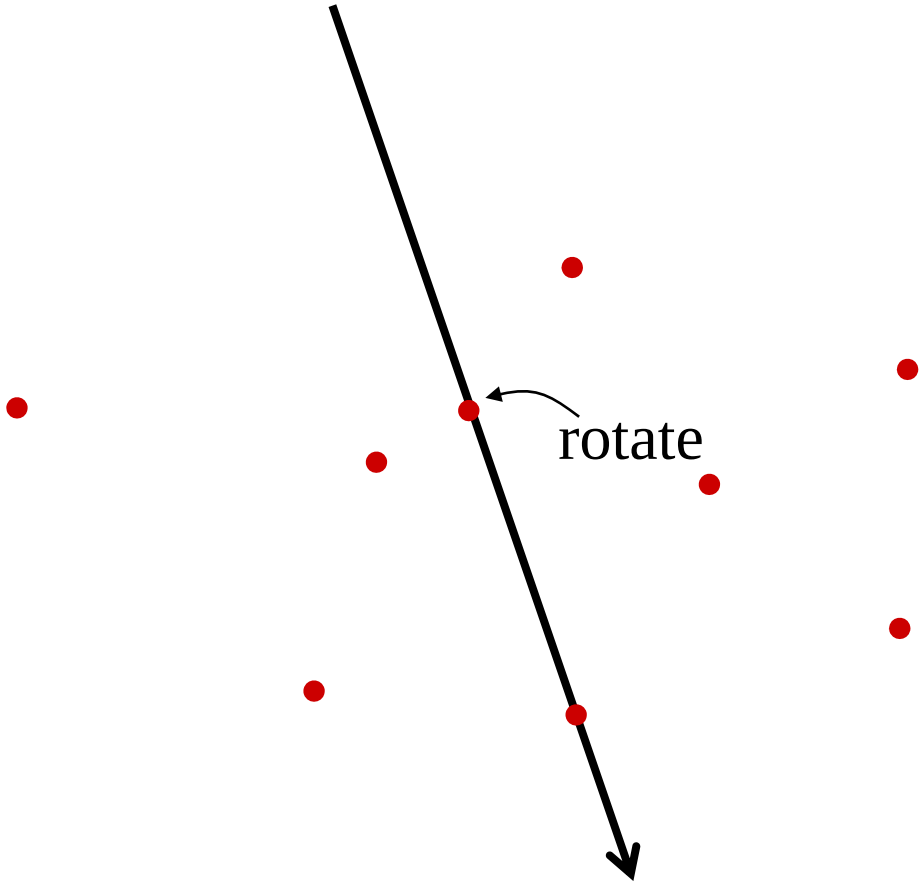
Left: 4
Right: 3

Rotation



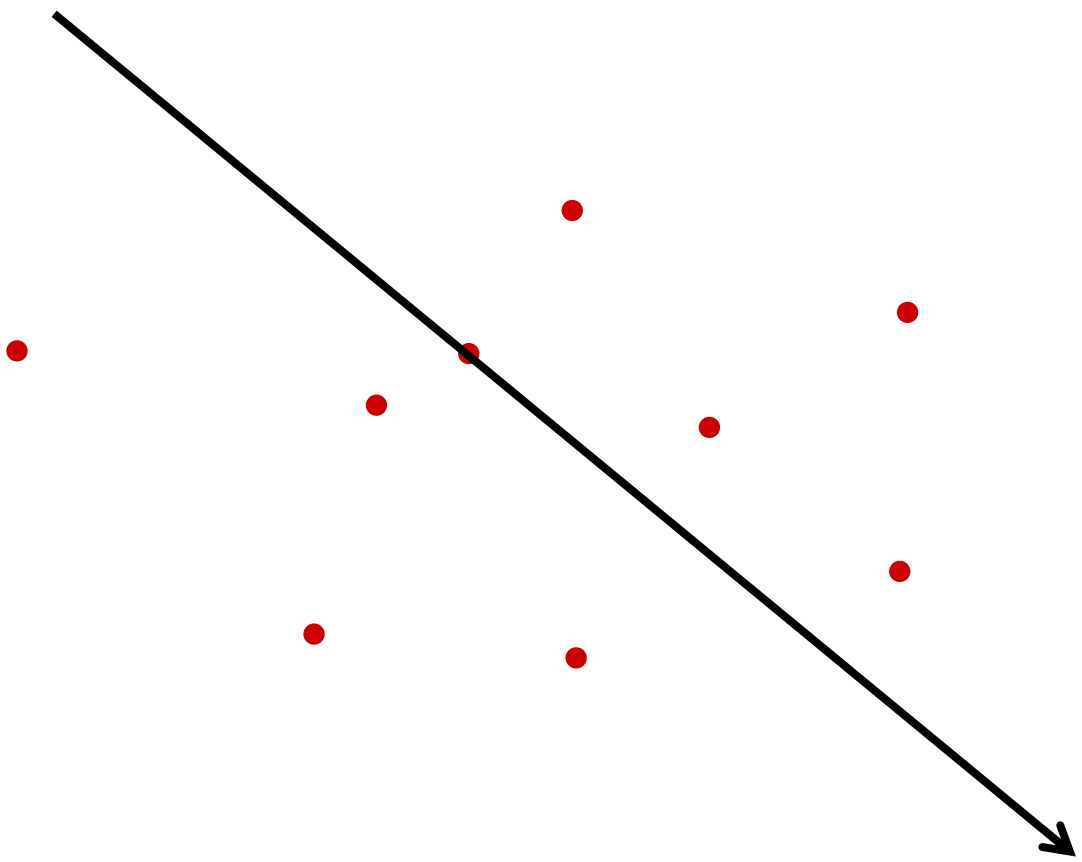
Left: 3
Right: 4

Rotation



Left: 4
Right: 3

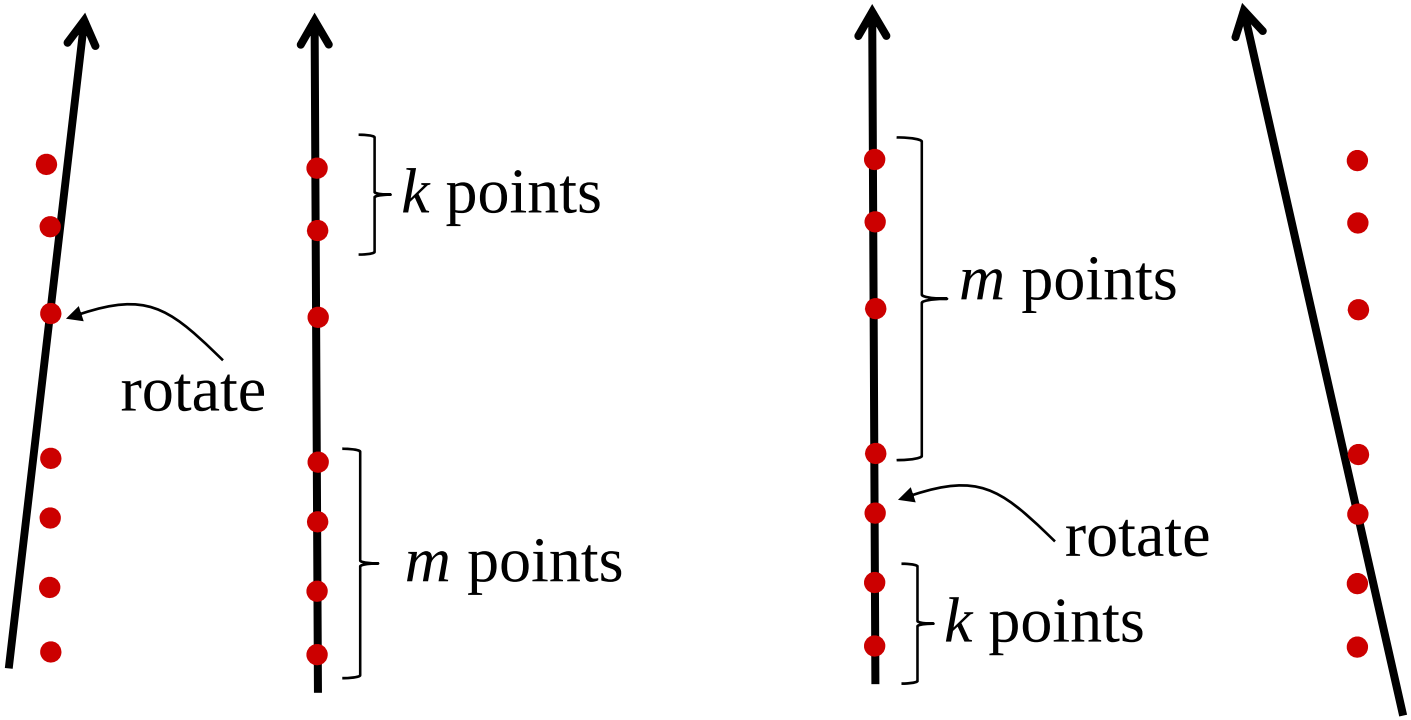
Rotation



Left: 4
Right: 4

Proof Continued

In general, choose the rotation point such that the number of points on each side of l does not change.



Proof Continued

Throughout the rotation, there are at most |P|/2 points on each side of *l*.

After 180° rotation, we get the line which is *l* but directed in the other direction.

Let *t* be the number of blue points to the left of *l* at the beginning. In the end, *t* points are on the right side of *l*, so |*Q*|-*t*-1 are on the left side. Therefore, there must have been one orientation of *l* such that there were *t* most |*Q*|/2 points on each side of *l*.