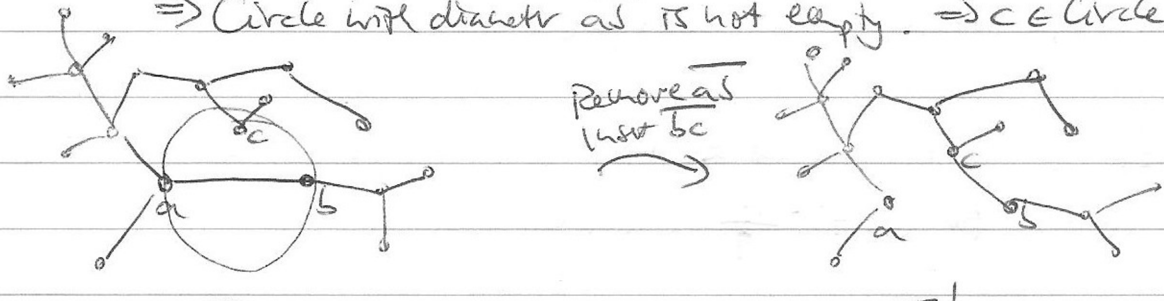
Euclidean MST and DT

- Given $P \subseteq \mathbb{R}^2$, $|P| = n$, let G be the Euclidean graph with $V = P$
  = complete (undirected) graph with edge weights being the Euclidean distance between incident vertices
- MST def. (of point set) Kruskal's algo: $O(|E| \log |E|) = O(n^2 \log n)$
  Prim's algo: $O(|E| + |V| \log |V|) = O(n^2 + n \log n)$

Theorem: The MST of a set of points P ~~(in any dimension)~~ is a subgraph of the Delaunay triangulation

Proof: Let T be MST for P, $w(T)$ its weight.
Let $\overline{ab}$ be an edge and assume, by contradiction, it is not Delaunay.
$\Rightarrow$ Circle with diameter $\overline{ab}$ is not empty. $\Rightarrow c \in$ Circle



T $\qquad\qquad$ T'

Since $\|\overline{cb}\| < \|\overline{ab}\|$ $\Rightarrow w(T') = w(T) \underbrace{- \|\overline{ab}\| + \|\overline{cb}\|}_{<0} < w(T)$ ↯

□

$\Rightarrow$ Compute MST on Delaunay triangulation (has $O(n)$ edges) using Kruskal's