CMPS 6640/4040 Computational Geometry – Spring 16

4/14/16

5. Homework

Due **3/31/16** before class

1. **Big-Oh Induction (10 points)**
   (a) (5 points) Consider the recurrence $T(n) = 8T(n/2) + n^2$. Use big-Oh induction to prove that $T(n) \in O(n^3)$.
   (Hint: You need to subtract a term from n^3. What term should that be?)
   (b) (5 points) Use big-Oh induction to prove that the storage complexity $M(n)$ for a two-level cutting tree is in $O(n^{2+\varepsilon})$.
   (Hint: Use the recurrence developed in class.)

   You do not need to prove the base case.

2. **Simplicial Partition (10 points)**
   Let S be a set of n points in the plane.
   (a) (6 points) Suppose that the points in S lie on a $\sqrt{n} \times \sqrt{n}$ grid. (Assume for simplicity that n is a square.) Let r be a parameter with $3 \leq r \leq n$. Explain how to draw a fine simplicial partition for S of size r and crossing number $O(\sqrt{r})$.
   (b) (4 points) Now suppose all points from S are collinear. Explain how to draw a fine simplicial partition for S of size r. What is the crossing number of your partition?

   Make sure to explain how the construction works for general values of n and r, but also give some example drawings.

3. **Inverse Range Counting (10 points)**
   Let T be a set of n triangles in the plane. An inverse range counting query asks to count the number of triangles from T containing a query point q.
   (a) Design a data structure for inverse range counting queries that uses roughly linear storage (for example, $O(n \log^c n)$ for some constant c). Analyze the amount of storage and the query time of your data structure.
   (b) Can you do better if you know that all triangles are disjoint?

4. **Cutting (10 points)**
   Let L be a set of n lines in the plane.
   (a) (5 points) Suppose that L consists of $\lfloor n/2 \rfloor$ vertical lines and $\lceil n/2 \rceil$ horizontal lines. Let r be a parameter with $1 \leq r \leq n$. Explain how to draw a $(1/r)$-cutting for L of size $O(r^2)$.
   (b) Now suppose all lines from L are vertical. Explain how to draw a $(1/r)$-cutting for L. What is the size of your cutting?

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5. **Largest Weight Queries (10 points)**
   Let S be a set of n points in the plane, each having a positive real weight associated with them. Describe two data structures for the following query problem: find the point in a query half-plane with the largest weight. One data structure should use linear storage, and the other data structure should have logarithmic query time. Analyze the amount of storage and the query time of both data structures.